The modified logarithmic Sobolev inequality for quantum spin systems: classical and commuting nearest neighbour interactions

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Munich Center for Quantum Science and Technology

Open quantum systems

Problem

Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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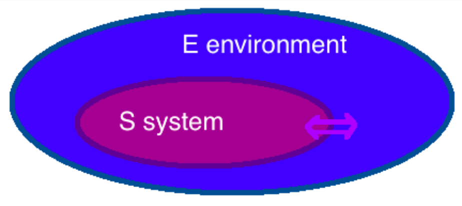

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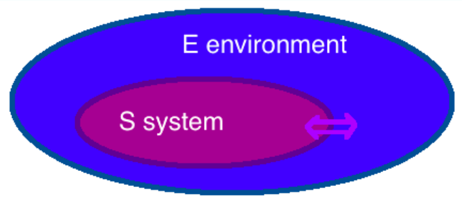

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- The continuous-time evolution of a state on S is given by a q. Markov semigroup (**Markovian approximation**).

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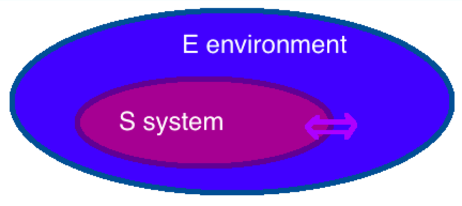

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Quantum Markov semigroups

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A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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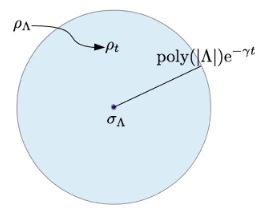

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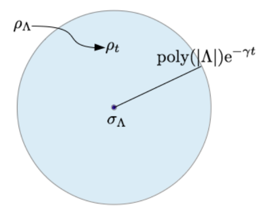

Modified logarithmic Sobolev inequality

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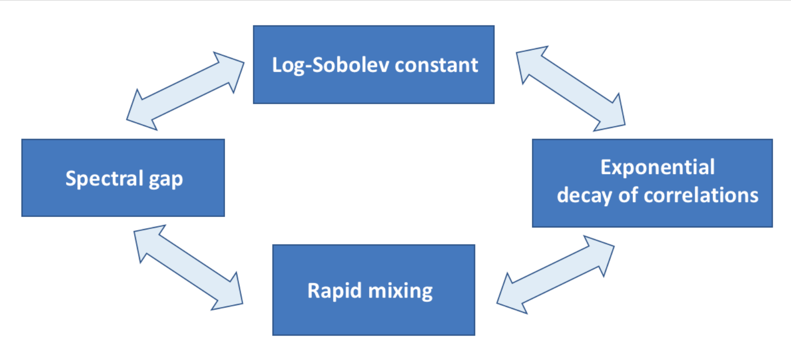
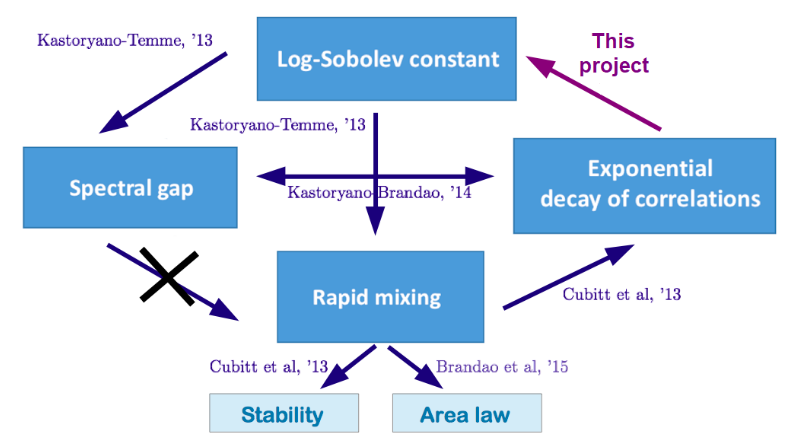
Classical spin systems



Quantum spin systems



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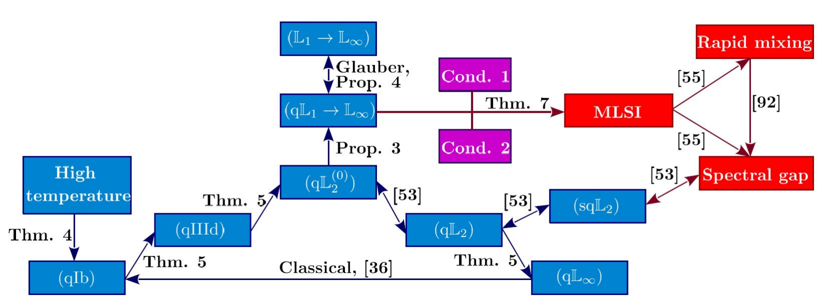

Main result

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Let H_Λ be a local commuting Hamiltonian such that one of the following conditions holds:

1. H_Λ is classical for $\beta < \beta_c$.
2. In 1D, H_Λ is a nearest neighbour Hamiltonian for $\beta < \beta_c$.

Then, there exists a local quantum Markov semigroup with fixed point σ_Λ, the Gibbs state of H_Λ, such that it has a positive **MLSI constant** which is independent of the system size.

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Modified logarithmic Sobolev inequality

Let $\left\{ \mathrm{e}^{t\mathcal{L}_\Lambda^*} \right\}_{t\geq 0}$ be a quantum Markov semigroup with $\mathcal{L}_\Lambda^*(\sigma_\Lambda) = 0$.

For $A \subset \Lambda$, let $E_A : \mathcal{B}(\mathcal{H}) \to \mathrm{Ker}(\mathcal{L}_A^*)$ be a conditional expectation, and

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Conditional modified logarithmic Sobolev inequality

For $A \subseteq \Lambda$, we say that a **conditional MLSI** on A holds for $\mathcal{L}_\Lambda^*$ if there exists a positive α_A such that for all $\rho_\Lambda \in \mathcal{S}_\Lambda$,

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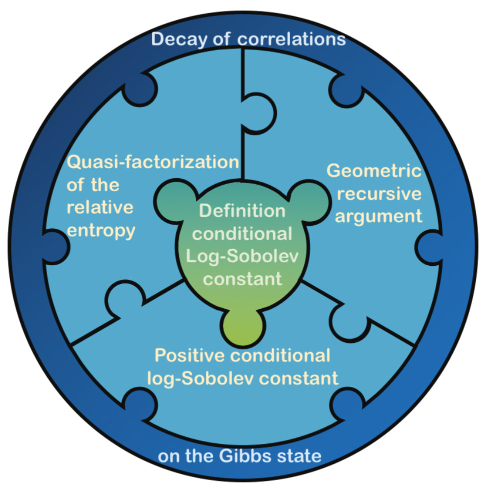
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Approximate tensorization of the relative entropy

In terms of the relative entropy, the **strong subadditivity of entropy** (Lieb-Ruskai '73) takes the form

$$D\left(\rho_{ABC} \middle\| \rho_B \otimes \frac{\mathbb{1}_{AC}}{d_{\mathcal{H}_{AC}}}\right) \leq D\left(\rho_{ABC} \middle\| \rho_{AB} \otimes \frac{\mathbb{1}_{C}}{d_{\mathcal{H}_{C}}}\right) + D\left(\rho_{ABC} \middle\| \rho_{BC} \otimes \frac{\mathbb{1}_{A}}{d_{\mathcal{H}_{A}}}\right).$$

Define $E_A := \lim_{t\to\infty} \mathrm{e}^{t\mathcal{L}_A^*}$. Then,

$$D(\rho\|E_{A\cup B}(\rho)) \leq D(\rho\|E_A(\rho))+D(\rho\|E_B(\rho)) \Leftrightarrow E_A\circ E_B = E_B\circ E_A = E_{A\cup B}$$

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In general, we present conditions in (Bardet-C.-Rouzé '20) for which

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Here, we show that a condition on the **fixed points** of the generator and a condition of **decay of correlations** imply

$$d = 0,\; c \sim 1 + \kappa\, \mathrm{e}^{-\mathrm{d}(\Lambda \setminus A, \Lambda\setminus B)} .$$

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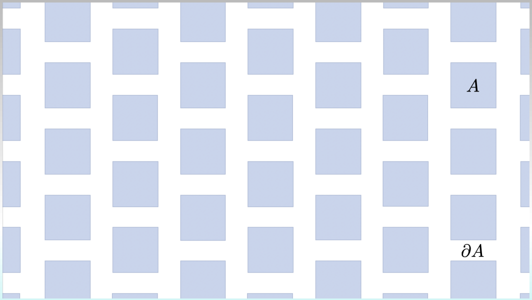
Peeling out



Chain rule for the relative entropy:

$$D(\rho_\Lambda \| \sigma_\Lambda) = D(\rho_\Lambda \| E_C(\rho_\Lambda)) + D(E_C(\rho_\Lambda) \| \sigma_\Lambda) \ .$$

For $D(E_C(\rho_\Lambda) \| \sigma_\Lambda)$, we use positivity of the complete MLSI (Junge-Gao-Laracuente '19)

$$\alpha_c := \inf_{k \in \mathbb{N}} \alpha \left(\mathcal{L}_\Lambda^* \otimes \mathbb{1}_k \right).$$

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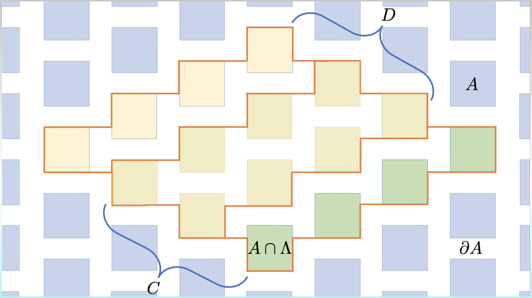

For $D(\rho_\Lambda \| E_C(\rho_\Lambda))$, we define a pinched MLSI

$$2\,\gamma_C\, D(E_A(\rho_\Lambda) || E_C \circ E_A(\rho_\Lambda)) \leq -\operatorname{tr}[\mathcal{L}_C^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \quad .$$

and apply the approximate tensorization result on such quantity.

Applications

- The output energy of an **Ising quantum annealer** subject to finite range classical thermal noise at high enough temperature outputs a state whose energy is close to that of the thermal state of the noise after an annealing time that is constant in system-size.
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