Exponential decay of mutual information for Gibbs states of local Hamiltonians

Ángela Capel (University of Tübingen)

Joint work with: **Andreas Bluhm** (U. Copenhagen)
Antonio Pérez-Hernández (UNED, Spain) .

Quantum, 6, 650, 2022

Quantum Information in Spain (ICE-7)
Granada, 24th May 2022

Collaborators

Andreas Bluhm

University of Copenhagen

Antonio Pérez-Hernández

Universidad Nacional de Educación a Distancia

Motivation

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

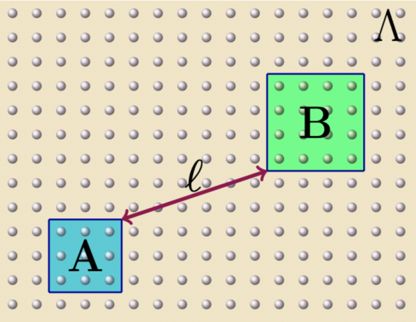
- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial C}$,
- **Gibbs state:** $\rho^\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} / \operatorname{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.

Motivation

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial C}$,
- **Gibbs state:** $\rho^\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} / \operatorname{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.



Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta(H_A+H_B)} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \ ?$$

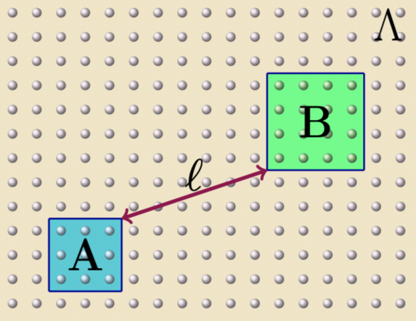
$$\operatorname{tr}_{A^c}[\rho^\Lambda] \otimes \operatorname{tr}_{B^c}[\rho^\Lambda] := \left(\rho^\Lambda\right)_A \otimes \left(\rho^\Lambda\right)_B \approx \operatorname{tr}_{(A\cup B)^c}[\rho^\Lambda] := \left(\rho^\Lambda\right)_{A\cup B} \ ?$$

Motivation

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial C}$,
- **Gibbs state:** $\rho^\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.



Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta(H_A+H_B)} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \ ?$$

$$\mathrm{tr}_{A^c}[\rho^\Lambda] \otimes \mathrm{tr}_{B^c}[\rho^\Lambda] := \left(\rho^\Lambda\right)_A \otimes \left(\rho^\Lambda\right)_B \approx \mathrm{tr}_{(A\cup B)^c}[\rho^\Lambda] := \left(\rho^\Lambda\right)_{A\cup B} \ ?$$

Overview

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

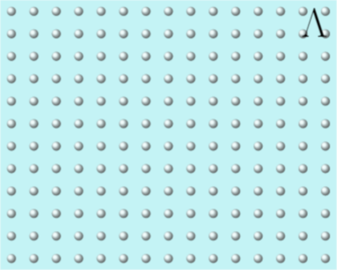
3 different forms of **decay of correlations**.

Overview

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

3 different forms of **decay of correlations**.



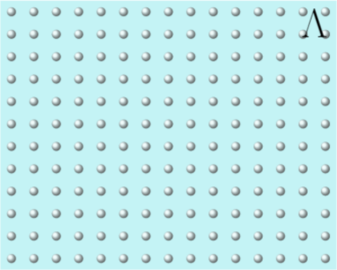
- A finite lattice $\Lambda \subset \mathbb{Z}^D$.
- For each site $x \in \Lambda$, $\mathcal{H}_x \equiv \mathbb{C}^d$.
- $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x \equiv (\mathbb{C}^d)^{\otimes |\Lambda|}$.
- $\mathfrak{A}_\Lambda = \mathcal{B}(\mathcal{H}_\Lambda) \equiv M_d(\mathbb{C})^{\otimes |\Lambda|}$.

Overview

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

3 different forms of **decay of correlations**.



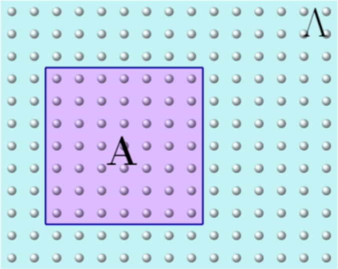
- A finite lattice $\Lambda \subset \mathbb{Z}^D$.
- For each site $x \in \Lambda$, $\mathcal{H}_x \equiv \mathbb{C}^d$.
- $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x \equiv (\mathbb{C}^d)^{\otimes |\Lambda|}$.
- $\mathfrak{A}_\Lambda = \mathcal{B}(\mathcal{H}_\Lambda) \equiv M_d(\mathbb{C})^{\otimes |\Lambda|}$.

Overview

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

3 different forms of **decay of correlations**.



- A finite lattice $\Lambda \subset \mathbb{Z}^D$.
- For each site $x \in \Lambda$, $\mathcal{H}_x \equiv \mathbb{C}^d$.
- $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x \equiv (\mathbb{C}^d)^{\otimes |\Lambda|}$.
- $\mathfrak{A}_\Lambda = \mathcal{B}(\mathcal{H}_\Lambda) \equiv M_d(\mathbb{C})^{\otimes |\Lambda|}$.
- $\mathfrak{A}_A \hookrightarrow \mathfrak{A}_\Lambda \;\; Q_A \mapsto Q_A \otimes \mathbb{1}_{\Lambda \setminus A}$.

Hamiltonian

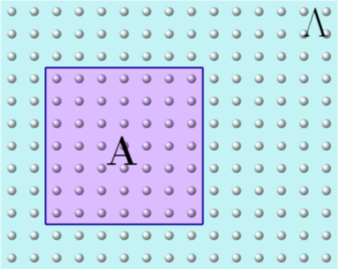
- $H_\Lambda = \sum_{X \subset \Lambda} H_X$, with $\|H_X\| \leq J$ and $\|H_X\| = 0$ if $\text{diam}(X) > r$.
- $\rho^\Lambda = \rho^\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} / \operatorname{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.

Overview

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

3 different forms of **decay of correlations**.

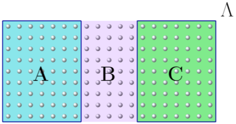


- A finite lattice $\Lambda \subset \mathbb{Z}^D$.
- For each site $x \in \Lambda$, $\mathcal{H}_x \equiv \mathbb{C}^d$.
- $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x \equiv (\mathbb{C}^d)^{\otimes |\Lambda|}$.
- $\mathfrak{A}_\Lambda = \mathcal{B}(\mathcal{H}_\Lambda) \equiv M_d(\mathbb{C})^{\otimes |\Lambda|}$.
- $\mathfrak{A}_A \hookrightarrow \mathfrak{A}_\Lambda \quad Q_A \mapsto Q_A \otimes \mathbb{1}_{\Lambda \setminus A}$.

Hamiltonian

- $H_\Lambda = \sum_{X \subset \Lambda} H_X$, with $\|H_X\| \leq J$ and $\|H_X\| = 0$ if $\text{diam}(X) > r$.
- $\rho^\Lambda = \rho^\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} \, / \, \text{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.

Overview



Operator correlation

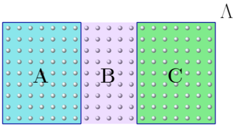
$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup\left|\Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C)\right|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Overview



Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\ Q \in \mathfrak{A}_X$$

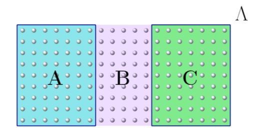
$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature	
		Low T	High T
Dimension	1 D	Exp. (~ Araki, '69)	
	Large D		

Overview



Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q] \,, \; Q \in \mathfrak{A}_X$$

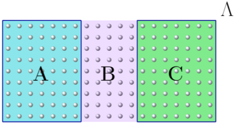
$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$

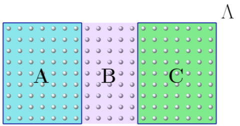
over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature	
		Low T	High T
Dimension	1 D		Exp. (Kliesch et al. '14)
	Large D		

Overview





Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Mutual information

$I_\rho(A:C) := D(\rho_{AC}||\rho_A \otimes \rho_C)$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

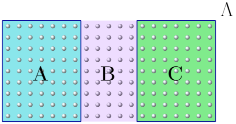
$$I_\rho(A:C) \geq \frac{1}{2}\,\mathrm{Corr}_\rho(A:C)^2$$

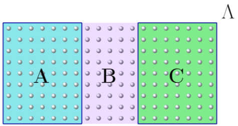
Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D		Exp. (Kliesch et al. '14)
Dimension	Large D		Exp. (Kliesch et al. '14)

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

Overview





Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\ Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Mutual information

$$I_\rho(A:C) := D(\rho_{AC} || \rho_A \otimes \rho_C)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

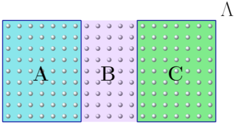
$$I_\rho(A:C) \geq \frac{1}{2}\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D		Exp. (Kliesch et al. '14)
Dimension	Large D		Exp. (Kliesch et al. '14)

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

Overview



Operator correlation

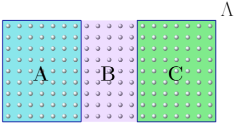
$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature	
		Low T	High T
Dimension	1 D		Exp. (Kliesch et al. '14)
	Large D		



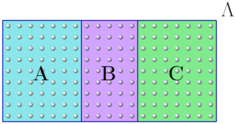
Mutual information

$$I_\rho(A:C) := D(\rho_{AC} || \rho_A \otimes \rho_C)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

$$I_\rho(A:C) \geq \frac{1}{2}\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$



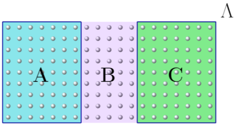
Conditional mutual inf.

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Overview



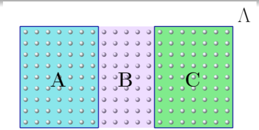
Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) :=$$
$$\sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$
over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D		Exp. (Kliesch et al. '14)
Dimension	Large D		Exp. (Kliesch et al. '14)



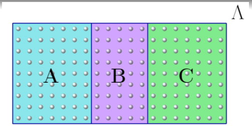
Mutual information

$I_\rho(A:C) := D(\rho_{AC} || \rho_A \otimes \rho_C)$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

$$I_\rho(A:C) \geq \frac{1}{2}\,\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$



Conditional mutual inf.

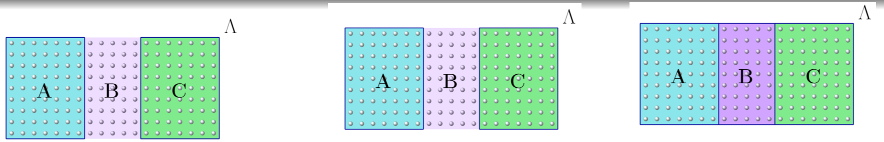
$I_\rho(A:C|B) := S(\rho_{AB})$
$+ S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$

for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Conditional Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (Kato-Brandao, '19)	Subexp. (Kato-Brandao, '19)
Dimension	Large D		

Overview



Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup |\Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C)|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D		Exp. (Kliesch et al. '14)
Dimension	Large D		Exp. (Kliesch et al. '14)

Mutual information

$$I_\rho(A:C) := D(\rho_{AC} \| \rho_A \otimes \rho_C)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

$$I_\rho(A:C) \geq \frac{1}{2}\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

Conditional mutual inf.

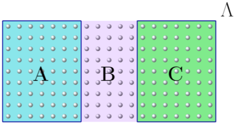
$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

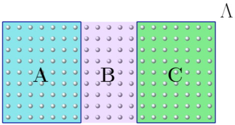
for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

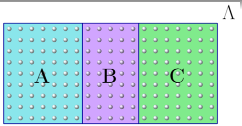
Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Conditional Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
Dimension	Large D		Exp. (Kuwahara et al. '20)

Overview







Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) :=$$
$$\sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$
over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D		Exp. (Kliesch et al. '14)
Dimension	Large D		(Exp. (Kliesch et al. '14) spans both rows)

Mutual information

$$I_\rho(A:C) := D(\rho_{AC} \| \rho_A \otimes \rho_C)$$
for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

$$I_\rho(A:C) \geq \frac{1}{2}\,\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (Kato-Brandao, '19)	(Subexp. (Kato-Brandao, '19) spans Low T and High T)
Dimension	Large D		

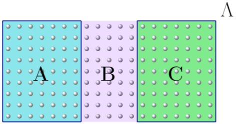
Conditional mutual inf.

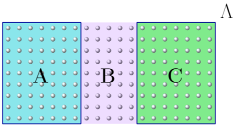
$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$
for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

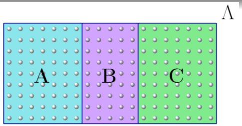
Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Conditional Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
Dimension	Large D		(Exp. (Kuwahara et al. '20) spans both rows)

Overview







Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) :=$$

$$\sup |\Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C)|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Mutual information

$$I_\rho(A:C) := D(\rho_{AC} || \rho_A \otimes \rho_C)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

Conditional mutual inf.

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho \log \rho]$

$$I_\rho(A:C) \geq \frac{1}{2}\,\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D		Exp. (Kliesch et al. '14)
Dimension	Large D		Exp. (Kliesch et al. '14)

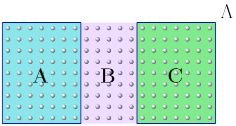
Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

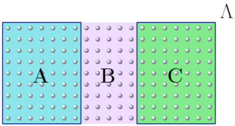
Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
Dimension	Large D		Exp. (Kuwahara et al. '20)

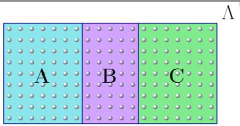
Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Conditional Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
Dimension	Large D		Exp. (Kuwahara et al. '20)

Overview







Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) :=$$
$$\sup |\Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C)|$$
$$\text{over } \|O_A\|, \|O_C\| \leq 1.$$

Mutual information

$$I_\rho(A:C) := D(\rho_{AC}\|\rho_A \otimes \rho_C)$$
for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

$$I_\rho(A:C) \geq \frac{1}{2}\mathrm{Corr}_\rho(A:C)^2$$

Conditional mutual inf.

$$I_\rho(A:C|B) := S(\rho_{AB})$$
$$+ S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$
for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature	
		Low T	High T
Dimension	1 D	Exponential	
	Large D		Exp. (Kliesch et al. '14)

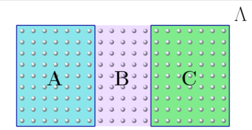
Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

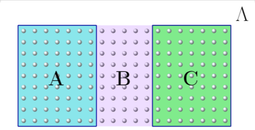
Mutual Information		Temperature	
		Low T	High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
	Large D		

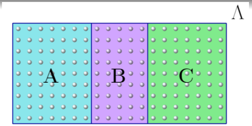
Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Conditional Mutual Information		Temperature	
		Low T	High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
	Large D		

Overview







Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup|\Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C)|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D	Exponential	Exponential
Dimension	Large D		Exp. (Kliesch et al. ’14)

Mutual information

$$I_\rho(A:C) := D(\rho_{AC}||\rho_A \otimes \rho_C)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

$$I_\rho(A:C) \geq \frac{1}{2}\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$I_{\rho^\Lambda}(A:C) \leq f'(\mathrm{d}(A:C))$

Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Exponential	Exponential
Dimension	Large D		Exp. (KKB ’20)

Conditional mutual inf.

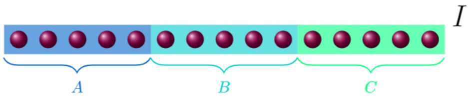
$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

Decay:
$I_{\rho^\Lambda}(A:C|B) \leq f''(\mathrm{d}(A:C))$

Conditional Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Subexp. (KB, ’19)	Exp. (Kuwahara et al. ’20)
Dimension	Large D		Exp. (Kuwahara et al. ’20)

Exponential decay of correlations



Operator correlation

$$\Psi_I(Q) := \mathrm{Tr}[\rho^I Q]\,,\ Q \in \mathfrak{A}_I \quad \text{with } \rho^I = \mathrm{e}^{-\beta H_I} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$$

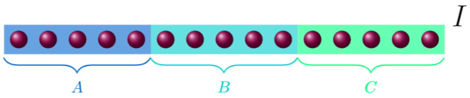
$$\boxed{\mathrm{Corr}_{\rho^I}(A:C) := \sup_{\|O_A\|,\|O_C\|\leq 1} \left| \Psi_I(O_A O_C) - \Psi_I(O_A)\Psi_I(O_C) \right|}$$

Exponential uniform clustering

There exist $\mathcal{K}, \gamma > 0$ such that for every finite lattice $\Lambda = ABC$,

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq \mathcal{K}\,\mathrm{e}^{-\gamma \mathrm{d}(A,C)}$$

Exponential decay of correlations



Operator correlation

$$\Psi_I(Q) := \mathrm{Tr}[\rho^I Q]\,,\; Q \in \mathfrak{A}_I \quad \text{with } \rho^I = \mathrm{e}^{-\beta H_I} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$$

$$\boxed{\mathrm{Corr}_{\rho^I}(A:C) := \sup_{\|O_A\|,\|O_C\|\leq 1} \left| \Psi_I(O_A O_C) - \Psi_I(O_A)\Psi_I(O_C) \right|}$$

Exponential uniform clustering

There exist $\mathcal{K}, \gamma > 0$ such that for every finite lattice $\Lambda = ABC$,

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq \mathcal{K}\,\mathrm{e}^{-\gamma \mathrm{d}(A,C)}$$

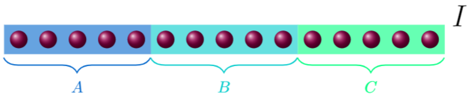
Conjecture:

In **1D**, there are no thermal phase transitions.

$\Downarrow$

Exponential uniform clustering holds in **1D** at every $\beta > 0$.

Exponential decay of correlations



Operator correlation

$$\Psi_I(Q) := \mathrm{Tr}[\rho^I Q]\,,\; Q \in \mathfrak{A}_I \quad \text{with } \rho^I = \mathrm{e}^{-\beta H_I} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$$

$$\mathrm{Corr}_{\rho^I}(A:C) := \sup_{\|O_A\|,\|O_C\|\leq 1} \left|\Psi_I(O_A O_C) - \Psi_I(O_A)\Psi_I(O_C)\right|$$

Exponential uniform clustering

There exist $\mathcal{K}, \gamma > 0$ such that for every finite lattice $\Lambda = ABC$,

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq \mathcal{K}\,\mathrm{e}^{-\gamma \mathrm{d}(A,C)}$$

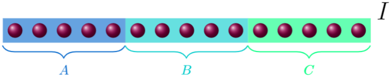
Conjecture:

In **1D**, there are no thermal phase transitions.

$\Downarrow$

Exponential uniform clustering holds in **1D** at every $\beta > 0$.

Exponential decay of correlations



Operator correlation

$$\Psi_I(Q) := \mathrm{Tr}[\rho^I Q]\,,\ Q \in \mathfrak{A}_I \quad \text{with } \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$$

$$\boxed{\mathrm{Corr}_{\rho^I}(A:C) := \sup_{\|O_A\|,\|O_C\|\leq 1} \left|\Psi_I(O_A O_C) - \Psi_I(O_A)\Psi_I(O_C)\right|}$$

Exponential uniform clustering in 1D

There exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$,

$$\mathrm{Corr}_{\rho^I}(A:C) \leq \mathcal{K}\,\mathrm{e}^{-\gamma|B|}$$

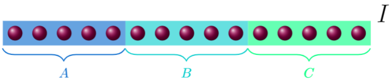
Theorem (Araki, '69)

- $\mathfrak{A}_{\mathbb{Z}}$ algebra of quasi-local observables.
- $\Psi_{\mathbb{Z}}$ unique KMS state $\quad \Psi_{\mathbb{Z}}(Q) = \lim_{I \nearrow \mathbb{Z}} \Psi_I(Q)$.

Assume translation-invariant interactions. There exist $\mathcal{K}', \gamma' > 0$ such that for every finite interval $I = ABC$,

$$\sup_{\|O_A\|,\|O_C\|\leq 1} \left|\Psi_{\mathbb{Z}}(O_A O_C) - \Psi_{\mathbb{Z}}(O_A)\Psi_{\mathbb{Z}}(O_C)\right| \leq \mathcal{K}'\,\mathrm{e}^{-\gamma'|B|}$$

Exponential decay of correlations



Operator correlation

$$\Psi_I(Q) := \mathrm{Tr}[\rho^I Q]\,,\; Q \in \mathfrak{A}_I \quad \text{with } \rho^I = \mathrm{e}^{-\beta H_I} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$$

$$\boxed{\mathrm{Corr}_{\rho^I}(A:C) := \sup_{\|O_A\|,\|O_C\|\leq 1} \left| \Psi_I(O_A O_C) - \Psi_I(O_A)\Psi_I(O_C) \right|}$$

Exponential uniform clustering in 1D

There exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$,

$$\mathrm{Corr}_{\rho^I}(A:C) \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|}$$

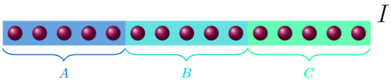
Theorem (Araki, '69)

- $\mathfrak{A}_{\mathbb{Z}}$ algebra of quasi-local observables.
- $\Psi_{\mathbb{Z}}$ unique KMS state $\quad \Psi_{\mathbb{Z}}(Q) = \lim_{I \nearrow \mathbb{Z}} \Psi_I(Q)$.

Assume translation-invariant interactions. There exist $\mathcal{K}', \gamma' > 0$ such that for every finite interval $I = ABC$,

$$\sup_{\|O_A\|,\|O_C\|\leq 1} \left| \Psi_{\mathbb{Z}}(O_A O_C) - \Psi_{\mathbb{Z}}(O_A)\Psi_{\mathbb{Z}}(O_C) \right| \leq \mathcal{K}'\, \mathrm{e}^{-\gamma' |B|}$$

Exponential decay of correlations



Operator correlation

$$\mathrm{Corr}_{\rho^I}(A:C) := \sup_{\|O_A\|,\|O_C\|\leq 1} \left|\Psi_I(O_A O_C) - \Psi_I(O_A)\Psi_I(O_C)\right|$$

Exponential uniform clustering in 1D

There exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$,

$$\mathrm{Corr}_{\rho^I}(A:C) \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|}$$

$\Downarrow$ (easy) $\Uparrow$ **Thm.** (Bluhm-C.-Perez Hernandez '21)

Theorem (Araki, '69)

- $\mathfrak{A}_{\mathbb{Z}}$ algebra of quasi-local observables.
- $\Psi_{\mathbb{Z}}$ unique KMS state $\Psi_{\mathbb{Z}}(Q) = \lim_{\mathbb{Z} \nearrow \mathbb{Z}} \Psi_I(Q)$.

Assume translation-invariant interactions. There exist $\mathcal{K}', \gamma' > 0$ such that for every finite interval $I = ABC$,

$$\sup_{\|O_A\|,\|O_C\|\leq 1} \left|\Psi_{\mathbb{Z}}(O_A O_C) - \Psi_{\mathbb{Z}}(O_A)\Psi_{\mathbb{Z}}(O_C)\right| \leq \mathcal{K}'\, \mathrm{e}^{-\gamma' |B|}$$

Decay of mutual information

Mutual information

$$I_\rho(A:B) := D(\rho_{AB} || \rho_A \otimes \rho_B) \,.$$

By Hölder and Pinsker's inequalities:

$$I_\rho(A:B) \geq \frac{1}{2} \| \rho_{AB} - \rho_A \otimes \rho_B \|_1^2 \geq \frac{1}{2} \operatorname{Corr}_\rho(A:B)^2 \,.$$

$$\text{Decay of mutual information} \quad \underset{\not\Leftarrow}{\Rightarrow} \quad \text{Decay of correlations}$$

There are states with small operator correlation and large mutual information in quantum data hiding (Hayden et al. '04).

Area laws

$$I_\rho(A:A^c) \leq O(|\partial A|) \,.$$

Wolf et al., '08: Area laws for the mutual information (improved: Kuwahara et al. '20).

Scalet et al., '21: Area laws for geometric Rényi divergences.

Decay of mutual information

Mutual information

$$I_\rho(A:B) := D(\rho_{AB} || \rho_A \otimes \rho_B) \, .$$

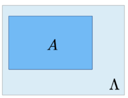
By Hölder and Pinsker's inequalities:

$$I_\rho(A:B) \geq \frac{1}{2} \| \rho_{AB} - \rho_A \otimes \rho_B \|_1^2 \geq \frac{1}{2} \operatorname{Corr}_\rho(A:B)^2 \, .$$

$$\text{Decay of mutual information} \; \underset{\not\Leftarrow}{\Rightarrow} \; \text{Decay of correlations}$$

There are states with small operator correlation and large mutual information in quantum data hiding (Hayden et al. '04).

Area laws



$$I_\rho(A:A^c) \leq O(|\partial A|) \, .$$

Wolf et al., '08: Area laws for the mutual information (improved: Kuwahara et al. '20).

Scalet et al., '21: Area laws for geometric Rényi divergences.

BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \operatorname{Tr}\left[\rho_{AC} \log \left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right] .$$

Geometric Rényi divergences, $\alpha > 1$

$$\widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) := \frac{1}{\alpha - 1} \log\operatorname{Tr}\left[\sigma_{AC}^{1/2} \left(\sigma_{AC}^{-1/2} \rho_{AC} \sigma_{AC}^{-1/2} \right)^{\alpha} \sigma_{AC}^{1/2} \right] .$$

BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \text{Tr}\left[\rho_{AC} \log \left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right] .$$

Geometric Rényi divergences, $\alpha > 1$

$$\widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) := \frac{1}{\alpha - 1} \log\text{Tr}\left[\sigma_{AC}^{1/2} \left(\sigma_{AC}^{-1/2} \rho_{AC} \sigma_{AC}^{-1/2} \right)^\alpha \sigma_{AC}^{1/2} \right] .$$

$$D(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) \quad \text{for every } \alpha > 1.$$

BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \operatorname{Tr}\left[\rho_{AC} \log\left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right].$$

Geometric Rényi divergences, $\alpha > 1$

$$\widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) := \frac{1}{\alpha - 1} \log\operatorname{Tr}\left[\sigma_{AC}^{1/2} \left(\sigma_{AC}^{-1/2} \rho_{AC} \sigma_{AC}^{-1/2} \right)^{\alpha} \sigma_{AC}^{1/2} \right].$$

$$D(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) \quad \text{for every } \alpha > 1.$$

BS-mutual information

$$\widehat{I}_\rho(A : C) := \widehat{D}(\rho_{AC} \,||\, \rho_A \otimes \rho_C)$$

BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \operatorname{Tr}\left[\rho_{AC} \log\left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right].$$

Geometric Rényi divergences, $\alpha > 1$

$$\widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) := \frac{1}{\alpha - 1} \log\operatorname{Tr}\left[\sigma_{AC}^{1/2} \left(\sigma_{AC}^{-1/2} \rho_{AC} \sigma_{AC}^{-1/2} \right)^{\alpha} \sigma_{AC}^{1/2} \right].$$

$$D(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) \quad \text{for every } \alpha > 1.$$

BS-mutual information

$$\widehat{I}_\rho(A:C) := \widehat{D}(\rho_{AC} \,||\, \rho_A \otimes \rho_C)$$

Bound for the BS-mutual information (Bluhm-C.-Pérez Hernández, '21)

$$\widehat{I}_\rho(A:C) \leq \left\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \right\|.$$

BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \operatorname{Tr}\left[\rho_{AC} \log\left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right].$$

Geometric Rényi divergences, $\alpha > 1$

$$\widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) := \frac{1}{\alpha - 1} \log\operatorname{Tr}\left[\sigma_{AC}^{1/2} \left(\sigma_{AC}^{-1/2} \rho_{AC} \sigma_{AC}^{-1/2} \right)^\alpha \sigma_{AC}^{1/2} \right].$$

$$D(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) \quad \text{for every } \alpha > 1.$$

BS-mutual information

$$\widehat{I}_\rho(A:C) := \widehat{D}(\rho_{AC} \,||\, \rho_A \otimes \rho_C)$$

Bound for the BS-mutual information (Bluhm-C.-Pérez Hernández, '21)

$$\widehat{I}_\rho(A:C) \leq \left\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \right\|.$$

Chain of inequalities:

$$\frac{1}{2} \operatorname{Corr}(A:C)^2 \leq \frac{1}{2} \|\rho_{AC} - \rho_A \otimes \rho_C\|_1^2 \leq I_\rho(A:C)$$

$$\leq \widehat{I}_\rho(A:C) \leq \widehat{I}_\rho^\alpha(A:C) \leq \|\rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC}\|.$$

BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \operatorname{Tr}\left[\rho_{AC} \log\left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right] .$$

Geometric Rényi divergences, $\alpha > 1$

$$\widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) := \frac{1}{\alpha - 1} \log\operatorname{Tr}\left[\sigma_{AC}^{1/2} \left(\sigma_{AC}^{-1/2} \rho_{AC} \sigma_{AC}^{-1/2} \right)^\alpha \sigma_{AC}^{1/2} \right] .$$

$$D(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) \leq \widehat{D}_\alpha(\rho_{AC} \,||\, \sigma_{AC}) \quad \text{for every } \alpha > 1.$$

BS-mutual information

$$\widehat{I}_\rho(A:C) := \widehat{D}(\rho_{AC} \,||\, \rho_A \otimes \rho_C)$$

Bound for the BS-mutual information (Bluhm-C.-Pérez Hernández, '21)

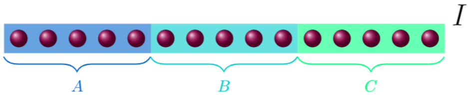
$$\widehat{I}_\rho(A:C) \leq \left\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \right\| .$$

Chain of inequalities:

$$\frac{1}{2} \operatorname{Corr}(A:C)^2 \leq \frac{1}{2} \| \rho_{AC} - \rho_A \otimes \rho_C \|_1^2 \leq I_\rho(A:C)$$

$$\leq \widehat{I}_\rho(A:C) \leq \widehat{I}_\rho^\alpha(A:C) \leq \| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| .$$

Exponential decay of mutual information in 1D



Exponential decay mixing condition (Bluhm-C.-Pérez Hernández, '21)

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

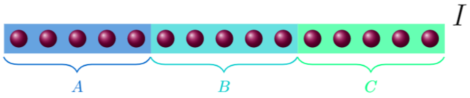
$$I_\rho(A:C) \leq \widehat{I}_\rho(A:C) \leq \| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|}$$

Corollary (Bluhm-C.-Pérez Hernández, '21)

The following are equivalent:

1. Exponential decay of correlations for the infinite-chain thermal states.
2. Exponential uniform clustering.
3. Exponential decay of the mutual information

Exponential decay of mutual information in 1D



Exponential decay mixing condition (Bluhm-C.-Pérez Hernández, '21)

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I}/\mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$I_\rho(A:C) \leq \widehat{I}_\rho(A:C) \leq \| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|}$$

Corollary (Bluhm-C.-Pérez Hernández, '21)

The following are equivalent:

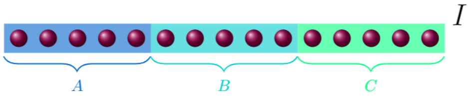
1. Exponential decay of correlations for the infinite-chain thermal states.
2. Exponential uniform clustering.
3. Exponential decay of the mutual information

Corollary (Bluhm-C.-Pérez Hernández, '21)

If ρ is the Gibbs state of a local, finite-range, translation-invariant (non-commuting) Hamiltonian, for any $\beta > 0$ and $I = ABC$,

$$I_\rho(A:C) \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|} \, .$$

Exponential decay of mutual information in 1D



Exponential decay mixing condition (Bluhm-C.-Pérez Hernández, '21)

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$I_\rho(A:C) \leq \widehat{I}_\rho(A:C) \leq \| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| \leq \mathcal{K} \, \mathrm{e}^{-\gamma |B|}$$

Corollary (Bluhm-C.-Pérez Hernández, '21)

The following are equivalent:

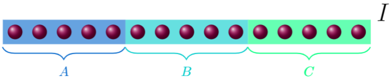
1. Exponential decay of correlations for the infinite-chain thermal states.
2. Exponential uniform clustering.
3. Exponential decay of the mutual information

Corollary (Bluhm-C.-Pérez Hernández, '21)

If ρ is the Gibbs state of a local, finite-range, translation-invariant (non-commuting) Hamiltonian, for any $\beta > 0$ and $I = ABC$,

$$I_\rho(A:C) \leq \mathcal{K} \, \mathrm{e}^{-\gamma |B|} \; .$$

Sketch of the proof

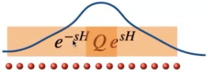


Exponential decay mixing condition

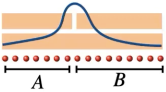
Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I}/\mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$I_\rho(A:C) \leq \widehat{I}_\rho(A:C) \leq \|\rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC}\| \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|}$$

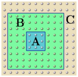
Ingredients:



- Imaginary-time **Lieb-Robinson bounds** (Araki, '69)



- Araki's **expansionals** (Araki, '69): $E = \mathrm{e}^{H_A + H_B} \mathrm{e}^{-H_{AB}}$
- **Exponential uniform clustering**



- **Local indistinguishability** (Brandao-Kast., '19): $\left(\rho^{ABC}\right)_A \approx \left(\rho^{AB}\right)_A$

Conditional Mutual Information

Conditional mutual information

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho \log \rho]$.

Strong subadditivity and recoverability (Lieb-Ruskai, '73, Petz, '86)

$$I_\rho(A:C|B) \geq 0$$

and $I_\rho(A:C|B) = 0$ if, and only if, ρ_{ABC} is a quantum Markov chain ($A \leftrightarrow B \leftrightarrow C$):

- $\rho_{ABC} = \rho_{AB}^{1/2}\rho_B^{-1/2}\rho_{BC}\rho_B^{-1/2}\rho_{AB}^{1/2}$.
- There is a recovery map $\mathcal{R}_{B\to AB}$ such that
$$\rho_{ABC} = \mathcal{R}_{B\to AB}(\rho_{BC})\,.$$

In general, for any quantum channel $\mathcal{T}$ and positive states ρ and σ,

$$D(\rho\|\sigma) \geq D(\mathcal{T}(\rho)\|\mathcal{T}(\sigma)) \text{ and } D(\rho\|\sigma) = D(\mathcal{T}(\rho)\|\mathcal{T}(\sigma)) \;\Leftrightarrow\; \rho = \sigma^{\frac{1}{2}}\mathcal{T}^*\left(\mathcal{T}(\sigma)^{-\frac{1}{2}}\mathcal{T}(\rho)\mathcal{T}(\sigma)^{-\frac{1}{2}}\right)\sigma^{\frac{1}{2}}$$

Conditional Mutual Information

Conditional mutual information

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho \log \rho]$.

Strong subadditivity and recoverability (Lieb-Ruskai, '73, Petz, '86)

$$I_\rho(A:C|B) \geq 0$$

and $I_\rho(A:C|B) = 0$ if, and only if, ρ_{ABC} is a quantum Markov chain ($A \leftrightarrow B \leftrightarrow C$):

- $\rho_{ABC} = \rho_{AB}^{1/2} \rho_B^{-1/2} \rho_{BC} \rho_B^{-1/2} \rho_{AB}^{1/2}$.
- There is a recovery map $\mathcal{R}_{B \to AB}$ such that
$$\rho_{ABC} = \mathcal{R}_{B \to AB}(\rho_{BC}) .$$

In general, for any quantum channel $\mathcal{T}$ and positive states ρ and σ,

$$D(\rho\|\sigma) \geq D(\mathcal{T}(\rho)\|\mathcal{T}(\sigma)) \text{ and } D(\rho\|\sigma) = D(\mathcal{T}(\rho)\|\mathcal{T}(\sigma)) \; \Leftrightarrow \; \rho = \sigma^{\frac{1}{2}} \mathcal{T}^* \left(\mathcal{T}(\sigma)^{-\frac{1}{2}} \mathcal{T}(\rho) \mathcal{T}(\sigma)^{-\frac{1}{2}} \right) \sigma^{\frac{1}{2}}$$

Decay CMI in 1D (Kato-Brandao, '19)

There exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

- $I_\rho(A:C|B) \leq \mathcal{K} \mathrm{e}^{-\gamma \sqrt{|B|}}$.
- There is a recovery map $\mathcal{R}_{B \to AB}$ such that
$$\left\| \rho_{ABC} - \mathcal{R}_{B \to AB}(\rho_{BC}) \right\|_1 \leq \mathrm{e}^{-\gamma \sqrt{|B|}} .$$

Conditional Mutual Information

Conditional mutual information

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho \log \rho]$.

Strong subadditivity and recoverability (Lieb-Ruskai, '73, Petz, '86)

$$I_\rho(A:C|B) \geq 0$$

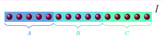
and $I_\rho(A:C|B) = 0$ if, and only if, ρ_{ABC} is a quantum Markov chain ($A \leftrightarrow B \leftrightarrow C$):

- $\rho_{ABC} = \rho_{AB}^{1/2} \rho_B^{-1/2} \rho_{BC} \rho_B^{-1/2} \rho_{AB}^{1/2}$.
- There is a recovery map $\mathcal{R}_{B \to AB}$ such that
$$\rho_{ABC} = \mathcal{R}_{B \to AB}(\rho_{BC}) \, .$$

In general, for any quantum channel $\mathcal{T}$ and positive states ρ and σ,

$$D(\rho \| \sigma) \geq D(\mathcal{T}(\rho) \| \mathcal{T}(\sigma)) \text{ and } D(\rho \| \sigma) = D(\mathcal{T}(\rho) \| \mathcal{T}(\sigma)) \;\Leftrightarrow\; \rho = \sigma^{\frac{1}{2}} \mathcal{T}^* \left(\mathcal{T}(\sigma)^{-\frac{1}{2}} \mathcal{T}(\rho) \mathcal{T}(\sigma)^{-\frac{1}{2}} \right) \sigma^{\frac{1}{2}}$$

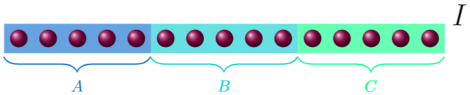
Decay CMI in 1D (Kato-Brandao, '19)



There exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

- $I_\rho(A:C|B) \leq \mathcal{K} \mathrm{e}^{-\gamma \sqrt{|B|}}$.
- There is a recovery map $\mathcal{R}_{B \to AB}$ such that
$$\left\| \rho_{ABC} - \mathcal{R}_{B \to AB}(\rho_{BC}) \right\|_1 \leq \mathrm{e}^{-\gamma \sqrt{|B|}} \, .$$

Approximate factorization of Gibbs states



BS - Conditional mutual information

$$\widehat{I}_\rho(A:C|B) := \widehat{D}(\rho_{ABC}||\rho_{AB}) - \widehat{D}(\rho_{BC}||\rho_B)$$

for $\widehat{D}(\rho||\sigma) = \mathrm{Tr}[\rho \log(\rho^{1/2}\sigma^{-1}\rho^{1/2})]$.

Bluhm-C., '20: For any quantum channel $\mathcal{T}$ and any positive states ρ and σ:

$$\rho = \sigma\, \mathcal{T}^* \left(\mathcal{T}(\sigma)^{-1}\mathcal{T}(\rho)\right) \quad \Longleftrightarrow \quad \widehat{D}(\rho||\sigma) = \widehat{D}(\mathcal{T}(\rho)||\mathcal{T}(\sigma)) .$$

For two positive states $\rho_{ABC}, \sigma_{ABC} \in \mathcal{H}_{ABC}$ such that $\sigma_{ABC} = \rho_{AB} \otimes \mathbb{1}_C/d_C$ and a $\mathcal{T} := \mathbb{1}_A/d_A \otimes \mathrm{Tr}_A$,

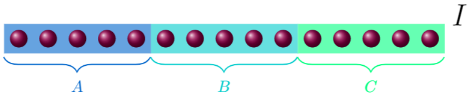
$$\rho_{ABC} = \rho_{AB}\rho_B^{-1}\rho_{BC} \quad \Longleftrightarrow \quad \widehat{D}(\rho_{ABC}||\rho_{AB}) = \widehat{D}(\rho_{BC}||\rho_B)$$

Approximate factorization (Bluhm-C.-Pérez Hernández, '21)

There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

$$\|\rho_{ABC} - \rho_{AB}\rho_B^{-1}\rho_{BC}\|_1 \leq \delta(|B|) .$$

Approximate factorization of Gibbs states



BS - Conditional mutual information

$$\widehat{I}_\rho(A:C|B) := \widehat{D}(\rho_{ABC}||\rho_{AB}) - \widehat{D}(\rho_{BC}||\rho_B)$$

for $\widehat{D}(\rho||\sigma) = \mathrm{Tr}[\rho \log(\rho^{1/2}\sigma^{-1}\rho^{1/2})]$.

Bluhm-C., '20: For any quantum channel $\mathcal{T}$ and any positive states ρ and σ:

$$\rho = \sigma\, \mathcal{T}^* \left(\mathcal{T}(\sigma)^{-1} \mathcal{T}(\rho) \right) \qquad \Longleftrightarrow \qquad \widehat{D}(\rho||\sigma) = \widehat{D}(\mathcal{T}(\rho)||\mathcal{T}(\sigma))\,.$$

For two positive states $\rho_{ABC}, \sigma_{ABC} \in \mathcal{H}_{ABC}$ such that $\sigma_{ABC} = \rho_{AB} \otimes \mathbb{1}_C/d_C$ and a $\mathcal{T} := \mathbb{1}_A/d_A \otimes \mathrm{Tr}_A$,

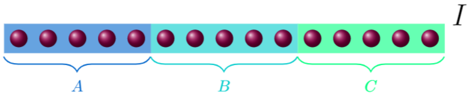
$$\rho_{ABC} = \rho_{AB}\rho_B^{-1}\rho_{BC} \qquad \Longleftrightarrow \qquad \widehat{D}(\rho_{ABC}||\rho_{AB}) = \widehat{D}(\rho_{BC}||\rho_B)$$

Approximate factorization (Bluhm-C.-Pérez Hernández, '21)

There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

$$\|\rho_{ABC} - \rho_{AB}\rho_B^{-1}\rho_{BC}\|_1 \leq \delta(|B|)\,.$$

Applications to MLSI



Exponential decay mixing condition

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| \leq \mathcal{K} \, \mathrm{e}^{-\gamma |B|}$$

MLSI for heat-bath dynamics (Bardet-C.-Lucia-Perez Garcia-Rouze, '21)

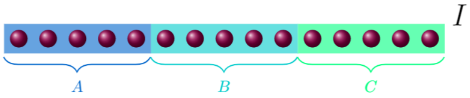
Theorem 7. *Let $\Lambda \subset\subset \mathbb{Z}$ be a finite chain. Let $\Phi : \Lambda \to \mathcal{A}_\Lambda$ be a k-local commuting potential and $H_\Lambda = \sum_{x \in \Lambda} \Phi(x)$ be its corresponding Hamiltonian, and denote by σ_Λ its Gibbs state. Let $\mathcal{L}_\Lambda^*$ be the generator of the heat-bath dynamics. Then, if Assumptions 1 and 2 hold, the MLSI constant of $\mathcal{L}_\Lambda^*$ is strictly positive and independent of $|\Lambda|$.*

Assumption 1 (mixing condition). *Let $\Lambda \subset\subset \mathbb{Z}$ be a finite chain, and let $C, D \subset \Lambda$ be the union of non-overlapping finite-sized segments of Λ. Let σ_Λ be the Gibbs state of a commuting Hamiltonian. The following inequality holds for certain positive constants K_1, K_2 independent of Λ, C, D:*

$$\left\| \sigma_C^{-1/2} \otimes \sigma_D^{-1/2} \, \sigma_{CD} \, \sigma_C^{-1/2} \otimes \sigma_D^{-1/2} - \mathbb{1}_{CD} \right\|_\infty \leq K_1 \mathrm{e}^{-K_2 d(C,D)},$$

where $d(C,D)$ is the distance between C and D, i.e., the minimum distance between two segments of C and D.

Applications to MLSI



Exponential decay mixing condition

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I}/\mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$\|\rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC}\| \leq \mathcal{K}\, \mathrm{e}^{-\gamma |B|}$$

MLSI for heat-bath dynamics (Bardet-C.-Lucia-Perez Garcia-Rouze, '21)

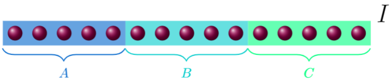
Theorem 7. *Let $\Lambda \subset\subset \mathbb{Z}$ be a finite chain. Let $\Phi : \Lambda \to \mathcal{A}_\Lambda$ be a k-local commuting potential and $H_\Lambda = \sum_{x \in \Lambda} \Phi(x)$ be its corresponding Hamiltonian, and denote by σ_Λ its Gibbs state. Let $\mathcal{L}_\Lambda^*$ be the generator of the heat-bath dynamics. Then, if Assumptions 1 and 2 hold, the MLSI constant of $\mathcal{L}_\Lambda^*$ is strictly positive and independent of $|\Lambda|$.*

Assumption 1 (mixing condition). *Let $\Lambda \subset\subset \mathbb{Z}$ be a finite chain, and let $C, D \subset \Lambda$ be the union of non-overlapping finite-sized segments of Λ. Let σ_Λ be the Gibbs state of a commuting Hamiltonian. The following inequality holds for certain positive constants K_1, K_2 independent of Λ, C, D:*

$$\left\| \sigma_C^{-1/2} \otimes \sigma_D^{-1/2} \, \sigma_{CD} \, \sigma_C^{-1/2} \otimes \sigma_D^{-1/2} - \mathbb{1}_{CD} \right\|_\infty \leq K_1 \mathrm{e}^{-K_2 d(C,D)},$$

where $d(C,D)$ is the distance between C and D, i.e., the minimum distance between two segments of C and D.

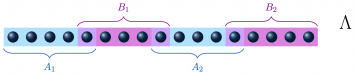
Applications to MLSI



Exponential decay mixing condition

For translation-invariant interactions, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| \leq \mathcal{K} \, \mathrm{e}^{-\gamma |B|}$$



MLSI for heat-bath dynamics (Bardet-C.-Gao-Lucia-Perez Garcia-Rouze, '21)

Positive MLSI ($O(\log(|I|))$) for Davies generators in 1D at any $\beta > 0$.

Theorem 3.1. *Let $\Lambda = [\![1, n]\!]$. For any $\beta > 0$, we denote by $\sigma \equiv \sigma^\beta$ the Gibbs state of a finite-range, translation-invariant, commuting Hamiltonian at inverse temperature $\beta > 0$. Consider $\mathcal{L}_{\Lambda *}^D$ the Davies generator of a quantum Markov semigroup $\{ e^{t \mathcal{L}_{\Lambda *}^D} \}_{t \geq 0}$ with unique fixed point σ. Then, there exists $\alpha_n = \Omega(\ln(n)^{-1})$ such that, for all $\rho \in \mathcal{D}(\mathcal{H}_\Lambda)$ and all $t \geq 0$,*

$$D(\rho_t \| \sigma) \leq e^{-\alpha_n t} D(\rho \| \sigma) \,, \qquad (25)$$

where $\rho_t := e^{t \mathcal{L}_{\Lambda *}^D}(\rho)$. Moreover, $\alpha_n = e^{-\mathcal{O}(\beta)}$ as a function of β.

Applications to MPO approximation

Approximate factorization (Bluhm-C.-Pérez Hernández, '21)

There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

$$\| \rho_{ABC} - \rho_{AB} \rho_B^{-1} \rho_{BC} \|_1 \leq \delta(|B|) \, .$$

Approximation by MPOs (Bluhm-C.-Pérez Hernández, '22)

There exists $\mathcal{G}, \mathcal{K} > 1$ such that for every finite interval $I \subset \mathbb{Z}$, split into $I = A_1 A_2 \dots A_n$ with $|A_j| = m$ for every $j = 1, \dots, n$, and for $\rho = \rho^I$ the Gibbs state on I,

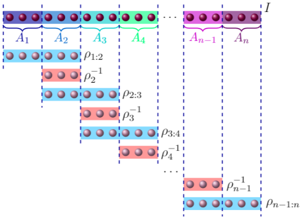
$$\left\| \rho - \rho_{1:2} \, \rho_2^{-1} \, \rho_{2:3} \, \rho_3^{-1} \, \rho_{3:4} \, \rho_4^{-1} \, \dots \, \rho_{n-1:n} \right\|_1 < \left(1 + \tilde{\mathcal{K}}(\beta) \frac{\mathcal{G}(\beta)^m}{m!} \right)^{n-2} - 1 \, .$$

Applications to MPO approximation

Approximate factorization (Bluhm-C.-Pérez Hernández, '21)

There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

$$\| \rho_{ABC} - \rho_{AB} \rho_B^{-1} \rho_{BC} \|_1 \leq \delta(|B|) \, .$$



Approximation by MPOs (Bluhm-C.-Pérez Hernández, '22)

There exists $\mathcal{G}, \mathcal{K} > 1$ such that for every finite interval $I \subset \mathbb{Z}$, split into $I = A_1 A_2 \ldots A_n$ with $|A_j| = m$ for every $j = 1, \ldots, n$, and for $\rho = \rho^I$ the Gibbs state on I,

$$\left\| \rho - \rho_{1:2} \, \rho_2^{-1} \, \rho_{2:3} \, \rho_3^{-1} \, \rho_{3:4} \, \rho_4^{-1} \, \ldots \, \rho_{n-1:n} \right\|_1 < \left(1 + \tilde{\mathcal{K}}(\beta) \frac{\mathcal{G}(\beta)^m}{m!} \right)^{n-2} - 1 \, .$$

Bond dimension: $\sim \log(m)/\text{loglog}(m)$.

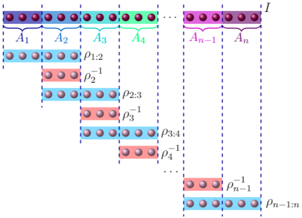
Kuwahara, Alhambra, Anshu, '21 $\sim \log(m)$.

Applications to MPO approximation

Approximate factorization (Bluhm-C.-Pérez Hernández, '21)

There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

$$\| \rho_{ABC} - \rho_{AB} \rho_B^{-1} \rho_{BC} \|_1 \leq \delta(|B|) \, .$$



Approximation by MPOs (Bluhm-C.-Pérez Hernández, '22)

There exists $\mathcal{G}, \mathcal{K} > 1$ such that for every finite interval $I \subset \mathbb{Z}$, split into $I = A_1 A_2 \ldots A_n$ with $|A_j| = m$ for every $j = 1, \ldots, n$, and for $\rho = \rho^I$ the Gibbs state on I,

$$\left\| \rho - \rho_{1:2} \, \rho_2^{-1} \, \rho_{2:3} \, \rho_3^{-1} \, \rho_{3:4} \, \rho_4^{-1} \, \ldots \, \rho_{n-1:n} \right\|_1 < \left(1 + \tilde{\mathcal{K}}(\beta) \frac{\mathcal{G}(\beta)^m}{m!} \right)^{n-2} - 1 \, .$$

Bond dimension: $\sim \log(m)/\text{loglog}(m)$.

Kuwahara, Alhambra, Anshu, '21 $\sim \log(m)$.

Conclusions

In this talk:

- For Gibbs states of local Hamiltonians in 1D at every $\beta > 0$, the following are equivalent:
  1. Exponential decay of correlations for the infinite-chain thermal states.
  2. Exponential uniform clustering.
  3. Exponential decay of the mutual information.

Conclusions

In this talk:

- For Gibbs states of local Hamiltonians in 1D at every $\beta > 0$, the following are equivalent:
  1. Exponential decay of correlations for the infinite-chain thermal states.
  2. Exponential uniform clustering.
  3. Exponential decay of the mutual information.

- Assuming translation-invariant interactions, all of them hold.

Conclusions

In this talk:

- For Gibbs states of local Hamiltonians in 1D at every $\beta > 0$, the following are equivalent:
  1. Exponential decay of correlations for the infinite-chain thermal states.
  2. Exponential uniform clustering.
  3. Exponential decay of the mutual information.
- Assuming translation-invariant interactions, all of them hold.
- Applications for positivity of MLSIs and approximation of Gibbs states by MPOs.

Conclusions

In this talk:

- For Gibbs states of local Hamiltonians in 1D at every $\beta > 0$, the following are equivalent:
  1. Exponential decay of correlations for the infinite-chain thermal states.
  2. Exponential uniform clustering.
  3. Exponential decay of the mutual information.
- Assuming translation-invariant interactions, all of them hold.
- Applications for positivity of MLSIs and approximation of Gibbs states by MPOs.

Open problems:

- More applications?
- Extension to higher dimensions?

Conclusions

In this talk:

- For Gibbs states of local Hamiltonians in 1D at every $\beta > 0$, the following are equivalent:
  1. Exponential decay of correlations for the infinite-chain thermal states.
  2. Exponential uniform clustering.
  3. Exponential decay of the mutual information.
- Assuming translation-invariant interactions, all of them hold.
- Applications for positivity of MLSIs and approximation of Gibbs states by MPOs.

Open problems:

- More applications?
- Extension to higher dimensions?

THANK YOU FOR YOUR ATTENTION!

Conclusions

In this talk:

- For Gibbs states of local Hamiltonians in 1D at every $\beta > 0$, the following are equivalent:
  1. Exponential decay of correlations for the infinite-chain thermal states.
  2. Exponential uniform clustering.
  3. Exponential decay of the mutual information.
- Assuming translation-invariant interactions, all of them hold.
- Applications for positivity of MLSIs and approximation of Gibbs states by MPOs.

Open problems:

- More applications?
- Extension to higher dimensions?

THANK YOU FOR YOUR ATTENTION!