Modified logarithmic Sobolev inequalities for quantum many-body systems

When does a quantum many-body system mix rapidly?

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Optimal Transport on Quantum Structures, Budapest
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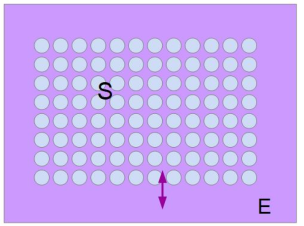
Motivation: Open quantum many-body systems

Open quantum many-body system.

No experiment can be executed at zero temperature or be completely shielded from noise.



- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x\in\Lambda} \mathcal{H}_x$.
- Density matrices: $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \; : \; \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

- Dynamics of S is dissipative!
- The continuous-time evolution of a state on S is given by a q. Markov semigroup (**Markovian approximation**).

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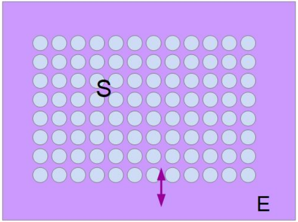

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Quantum Markov semigroup / Dissipative quantum evolution

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QMS generator

The infinitesimal generator $\mathcal{L}_\Lambda^*$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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For $\rho_\Lambda \in \mathcal{S}_\Lambda, \quad \mathcal{L}_\Lambda^*(\rho_\Lambda) = -i[H_\Lambda, \rho_\Lambda] + \sum_{k\in\Lambda} \widetilde{\mathcal{L}}_k^*(\rho_\Lambda)$ **GKLS equation.**

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We also assume that the quantum Markov process studied is **reversible**, i.e., it satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}_\Lambda(g) \rangle_\sigma = \langle \mathcal{L}_\Lambda(f), g \rangle_\sigma ,$$

for every $f, g \in \mathcal{B}_\Lambda$ and Hermitian, where

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Mixing time

- Under the previous conditions, there is always convergence to σ_Λ.
- How fast does convergence happen?

Note $\mathcal{T}_\infty^*(\rho) := \sigma_\Lambda$ for every ρ.

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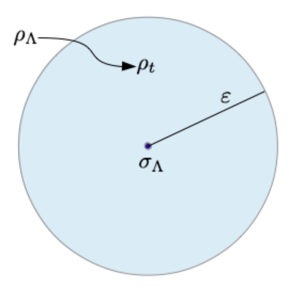
We define the **mixing time** of $\{\mathcal{T}_t^*\}$ by

$$t_{\text{mix}}(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \| \mathcal{T}_t^*(\rho) - \mathcal{T}_\infty^*(\rho) \|_1 \leq \varepsilon \right\}.$$



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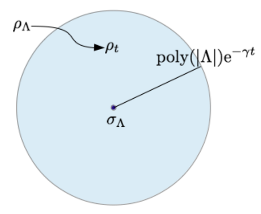
$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

$t_{\text{mix}}(\varepsilon) \sim \text{poly} \log(|\Lambda|)$.



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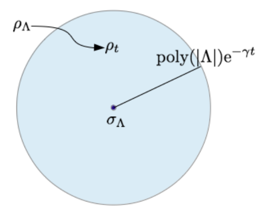

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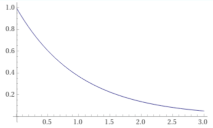

Lower bound for the derivative of $D(\rho_t||\sigma_\Lambda)$ in terms of itself:

$$2\alpha D(\rho_t||\sigma_\Lambda) \leq -\mathrm{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)].$$

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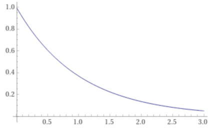

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If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) > 0$:

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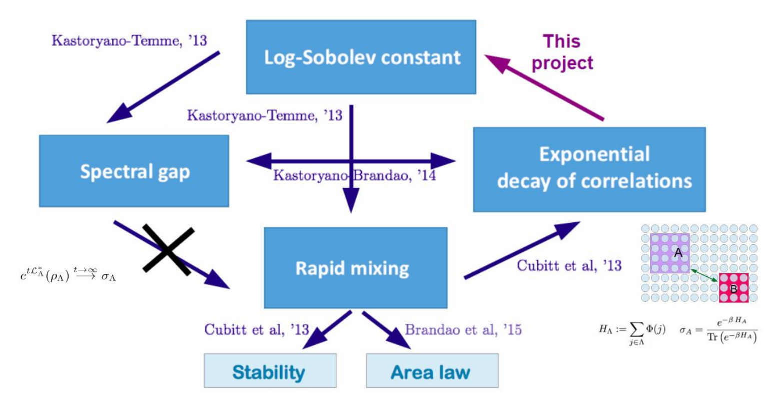
Quantum spin systems



Exp. decay of correlations:

$$\sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]| \leq K\,\mathrm{e}^{-\gamma d(A,B)} \quad .$$

Decay of correlations on Gibbs state

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial B}$,
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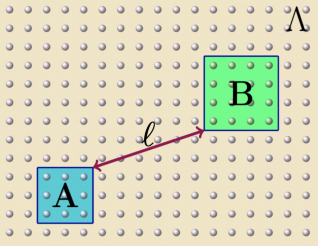

$\ell := \mathrm{dist}(A, B)$

Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta H_{A\cup B}} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \ ?$$

$$\mathrm{tr}_{A^c}[\sigma_\Lambda] \otimes \mathrm{tr}_{B^c}[\sigma_\Lambda] := (\sigma_\Lambda)_A \otimes (\sigma_\Lambda)_B \approx \mathrm{tr}_{(A\cup B)^c}[\sigma_\Lambda] := (\sigma_\Lambda)_{A\cup B} \ ?$$

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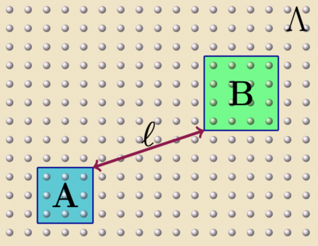

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3 different forms of **decay of correlations**.

Operator correlation

$$\mathrm{Corr}_\sigma(A:B) := \sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]|$$

Mutual information

$$I_\sigma(A:B) := D(\rho_{AB}||\rho_A \otimes \rho_B)$$

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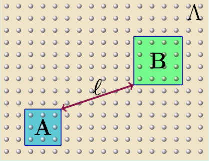
Mixing condition

$$\|h(\sigma_{AB})\|_\infty = \left\|\sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB}\right\|_\infty$$



Relation:

$$\frac{1}{2}\text{Corr}_\sigma(A:B)^2 \leq I_\sigma(A:B)$$

$$\leq \left\|\sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB}\right\|_\infty .$$

Decay of correlations on Gibbs state

3 different forms of **decay of correlations**.

Operator correlation

$$\mathrm{Corr}_\sigma(A:B) := \sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]|$$

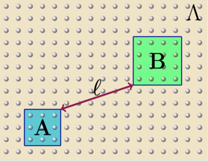
Mutual information

$$I_\sigma(A:B) := D(\rho_{AB} || \rho_A \otimes \rho_B)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

Mixing condition

$$\|h(\sigma_{AB})\|_\infty = \left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty$$



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Quantum spin systems

Thermalization

MLSI (log-Sobolev)

$$D(\rho_t||\sigma_\Lambda) \leq D(\rho_\Lambda||\sigma_\Lambda)e^{-2\,\alpha(\mathcal{L}_\Lambda^*)\,t}$$

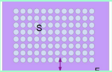
Rapid Mixing

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|)e^{-\gamma t}$$

Spectral gap

$$\text{Var}(\rho_t, \sigma_\Lambda) \leq \text{Var}(\rho_\Lambda, \sigma_\Lambda)e^{-\lambda(\mathcal{L}_\Lambda^*)\,t}$$

$$e^{t\mathcal{L}_\Lambda^*}(\rho_\Lambda) \xrightarrow{t\to\infty} \sigma_\Lambda$$



Decay of correlations

Mixing condition

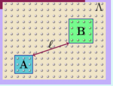
$$\left\| \sigma_A^{-1} \otimes \sigma_B^{-1}\sigma_{AB} - \mathbb{1}_{AB} \right\|_\infty \leq Ke^{-\gamma\, d(A,B)}$$

Mutual information

$$I_\sigma(A:B) \leq Ke^{-\gamma\, d(A,B)}$$

Operator correlation

$$\text{Corr}_\sigma(A:B) \leq Ke^{-\gamma\, d(A,B)}$$



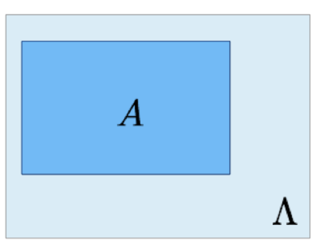
Objective

MLSI constant

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|\Lambda|) > 0.$$



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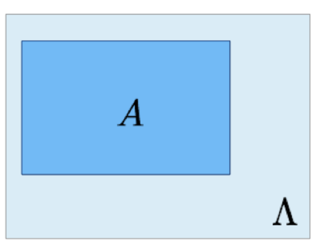

Can we prove something like

$$\alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|A|)\, \alpha(\mathcal{L}_A^*) > 0 \text{ ?}$$

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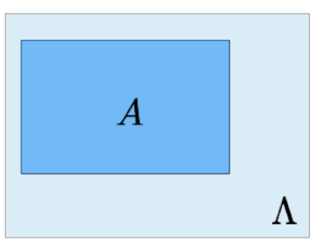

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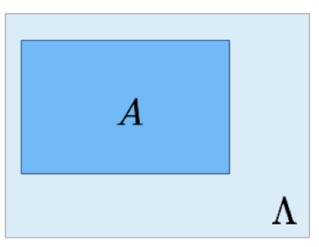

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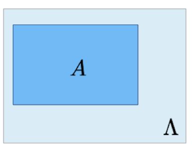
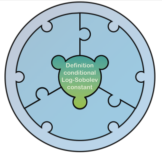
Conditional MLSI constant





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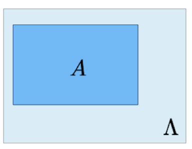

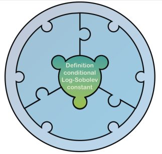

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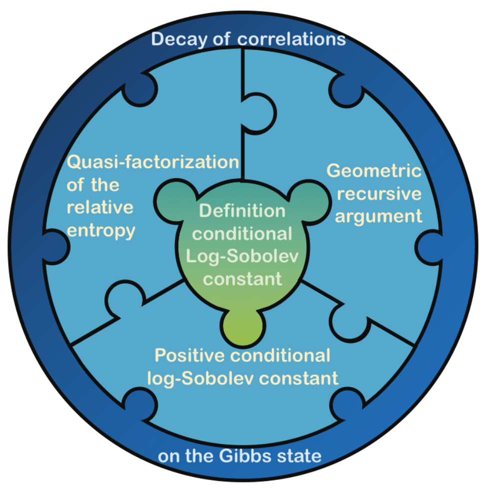
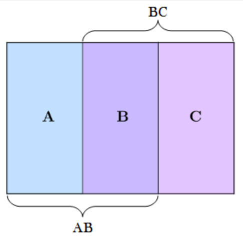
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

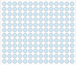
Example: Tensor product fixed point

(C.-Lucia-Pérez García '18)

(Beigi-Datta-Rouzé '18)

$$\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda) \qquad \textbf{heat-bath}$$

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x ,$$

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq$$

$$\leq \sum_{x \in \Lambda} D_x(\rho_\Lambda \| \sigma_\Lambda)$$

$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 D_x(\rho_\Lambda \| \sigma_\Lambda)}$

$$\leq \sum_{x \in \Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x^*)}$$

$$\leq \frac{1}{2 \inf_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x \in \Lambda} -\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]$$

$$= \frac{1}{2 \inf_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right)$$

$$\leq \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right).$$

Dynamics

Let $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}\left[e^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

$$\mathcal{L}_\Lambda^{H;*}(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right)$$

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Davies generator

The **Davies generator** is given by:

$$\mathcal{L}_\Lambda^D(X) := i[H_\Lambda, X] + \sum_{x\in\Lambda} \widetilde{\mathcal{L}}_x^D(X),$$

where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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The **Schmidt generator** (Bravyi-Vyalyi '05) can be written as:

$$\mathcal{L}_\Lambda^S(X) = \sum_{x\in\Lambda} \left(E_x^S(X) - X \right) ,$$

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Previous results

Let us recall: For $\alpha(\mathcal{L}_\Lambda^*)$ a MLSI constant,

$$\left\|\rho_t - \sigma_\Lambda\right\|_1 \leq \sqrt{2\log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t}.$$

Using the spectral gap $\lambda(\mathcal{L}_\Lambda^*)$:

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Let $\mathcal{L}_\Lambda^{H,D;*}$ be the **heat-bath** or **Davies** generator in 1D. Then, $\mathcal{L}_\Lambda^{H,D;*}$ has a positive spectral gap that is independent of the system size, for every temperature.

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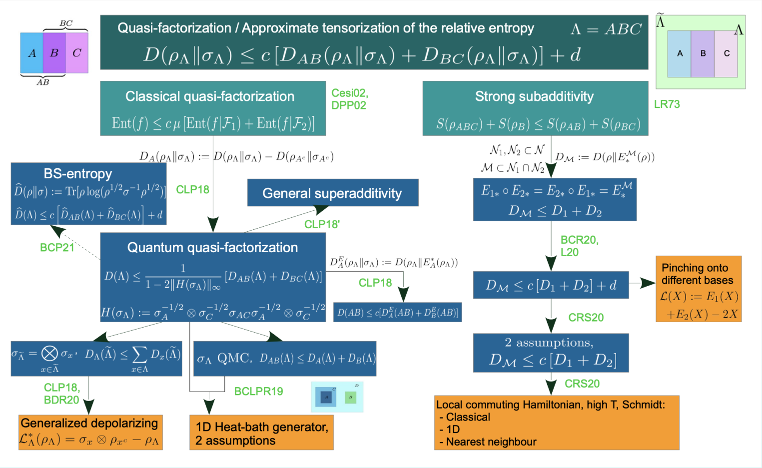
Quasi-factorization of the relative entropy

Results of **Quasi-Factorization** or **Approximate Tensorization**

Results of **Modified Logarithmic Sobolev Inequality**



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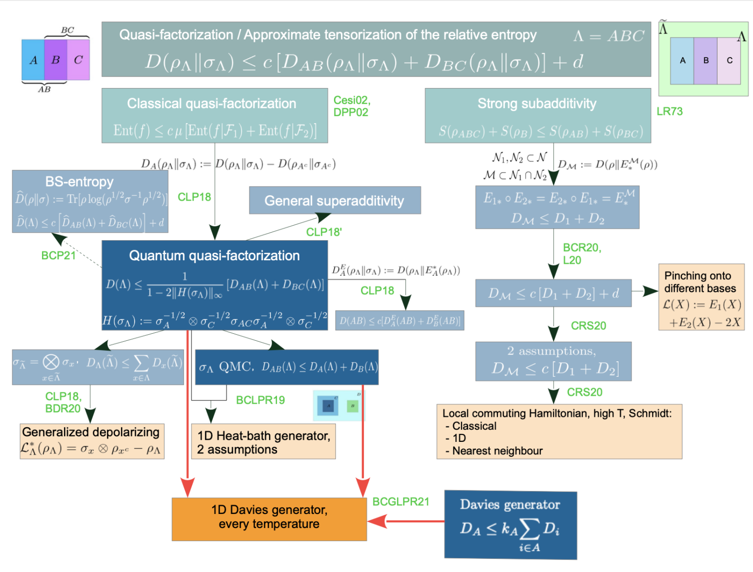

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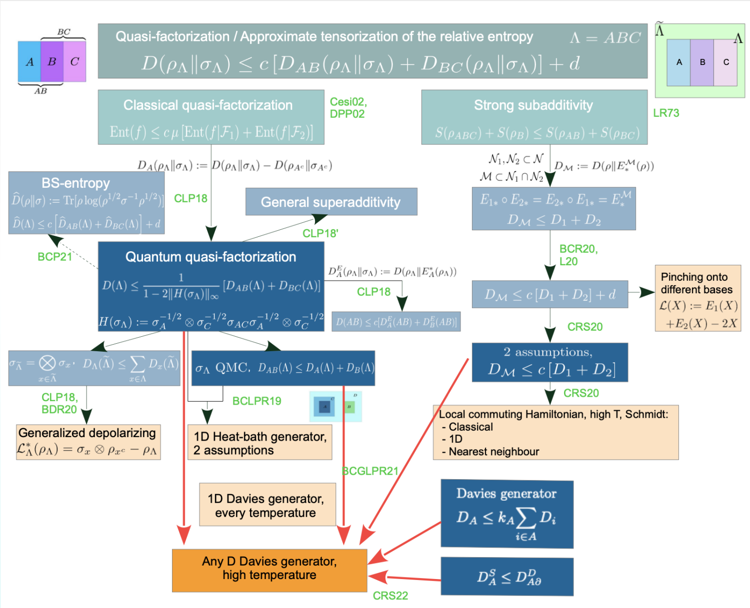

MLSI for Davies generators in 1D

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Let $\mathcal{L}_\Lambda^{D;*}$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^{D;*}$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^{D;*}) = \Omega(\ln(|\Lambda|)^{-1})$.

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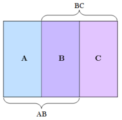
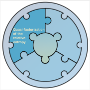
Proof: Conditional relative entropies + Quasi-factorization





Conditional relative entropies: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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Quasi-factorization (C.-Lucia-Pérez García '18)

Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

$$D(\rho_{ABC} \| \sigma_{ABC}) \leq \xi(\sigma_{AC}) \left[D_{AB}(\rho_{ABC} \| \sigma_{ABC}) + D_{BC}(\rho_{ABC} \| \sigma_{ABC}) \right],$$

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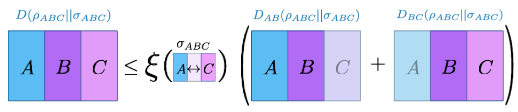
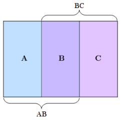
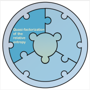


Proof: Conditional relative entropies + Quasi-factorization





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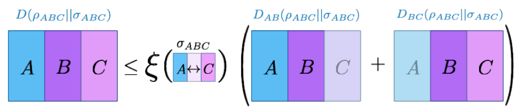

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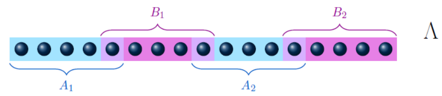

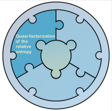

$\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}(e^{-\beta H_\Lambda})}$ is the Gibbs state of a k-local, commuting Hamiltonian H_Λ.

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Quasi-factorization for quantum Markov chains

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Since σ_Λ is a QMC between $A_i \leftrightarrow \partial(A_i) \leftrightarrow (A_i \cup \partial A_i)^c$, then:

$$D_A(\rho_\Lambda || \sigma_\Lambda) \leq \sum_i D_{A_i}(\rho_\Lambda || \sigma_\Lambda).$$

$$\sigma_\Lambda = \bigoplus_{j \in J} \sigma_{A_i (\partial a_i)_j^L} \otimes \sigma_{(\partial a_i)_j^R (A_i \cup \partial A_i)^c}$$

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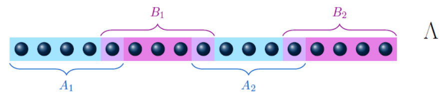

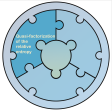

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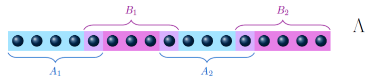

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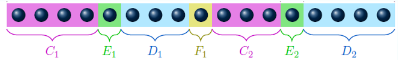

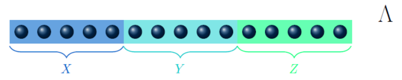

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Let σ_{XYZ} be the Gibbs state of a finite-range, translation-invariant Hamiltonian. There is $\ell \mapsto \delta(\ell)$ with exponential decay such that:

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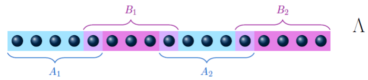

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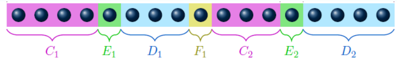

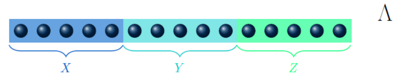

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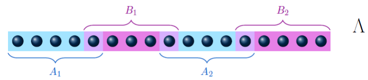

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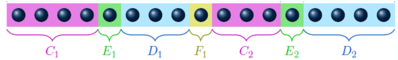

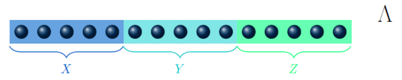

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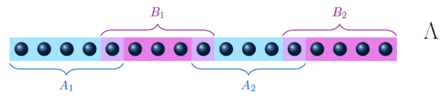
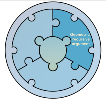
Proof: Geometric recursive argument





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and thus $\alpha(\mathcal{L}_\Lambda^{H;*}) \geq \dfrac{K}{\xi(\sigma_{A^c B^c})} \min \left\{ \alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}), \alpha_{B_i}(\mathcal{L}_\Lambda^{H;*}) \right\}$,

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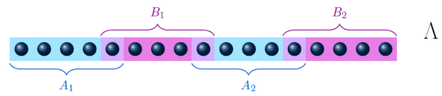

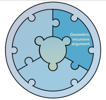

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Reduction of cond. relative entropies (Gao-Rouzé, '21)

$$D(\rho_\Lambda \| E^*_{A_i}(\rho_\Lambda)) \leq 4 k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E^*_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda} \, ,$$

where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

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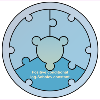
CMLSI (Gao-Rouzé, '21)

The CMLSI of the local generators is positive:

$$\alpha_c(\mathcal{L}_j^{D;*}) := \inf_{k \in \mathbb{N}} \alpha(\mathcal{L}_j^{D;*} \otimes \mathrm{Id}_k) > 0 \, .$$



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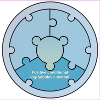

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The conditional expectations associated to Davies and heat-bath dynamics coincide.

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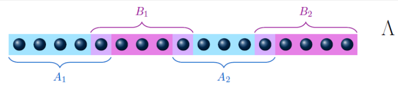
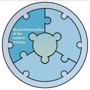
Sketch of the proof: Quasi-factorization





$\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}(e^{-\beta H_\Lambda})}$ is the Gibbs state of a k-local, commuting Hamiltonian H_Λ.

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Let $A \cup B = \Lambda \subset \mathbb{Z}$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$. The following holds

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \xi(\sigma_{A^c B^c}) \left[D_A(\rho_\Lambda || \sigma_\Lambda) + D_B(\rho_\Lambda || \sigma_\Lambda) \right],$$

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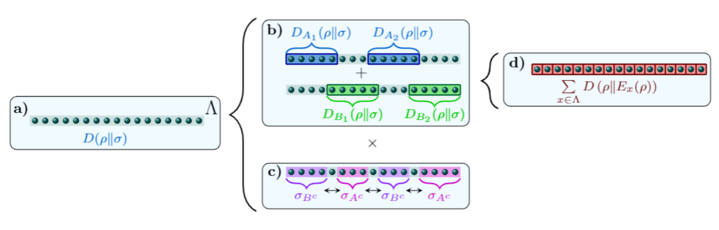

Last step: Spectral gap $\overset{\mathcal{O}(\log n)}{\mapsto}$ MLSI.

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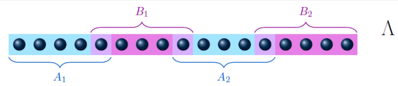

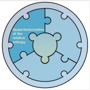

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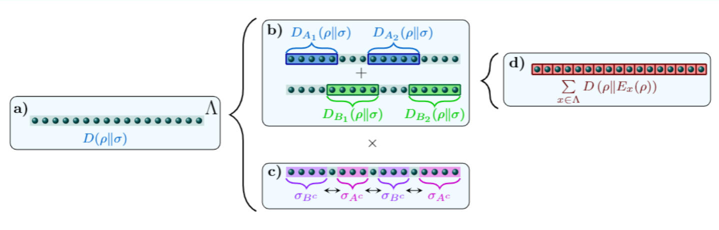

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Ángela Capel Cuevas (Universität Tübingen)

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We're hiring!

We're looking for candidates to fill the following positions within the **CRC "Mathematics of Many-Body Quantum Systems and Their Collective Phenomena" (UT-LMU-TUM):**

- 1 **PhD student** in co-supervision with Cambyse Rouzé (T. U. Munich).
- 1 **postdoc** in co-supervision with Stefan Teufel (U. Tübingen).

The positions could start on **January 1st, 2023**.

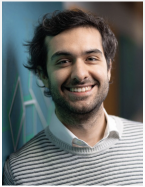
If you're interested, please get in touch! (or send an email to angela.capel@uni-tuebingen.de)

Thank you for your attention!

Do you have any questions?

David Pérez-García
U. Complutense
Madrid

Angelo Lucia
U. Complutense
Madrid



Cambyse Rouzé
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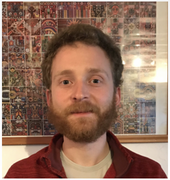

Ivan Bardet
Inria Paris

Daniel Stilck Franca
ENS Lyon

Antonio
Pérez-Hernández
UNED Madrid

Andreas Bluhm
U. Copenhagen

Li Gao
U. Houston