Spectral gap implies rapid mixing for commuting Hamiltonians

Modified logarithmic Sobolev inequalities for quantum many-body systems

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Joint work with **A. Alhambra, J. Kochanowski and C. Rouzé**

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Motivation: Open quantum many-body systems

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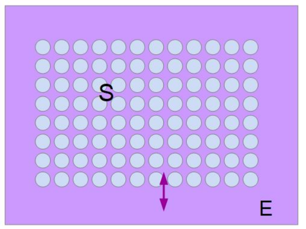
No experiment can be executed at zero temperature or be completely shielded from noise.



- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$.
- Density matrices: $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \,:\, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

- Dynamics of S is dissipative!
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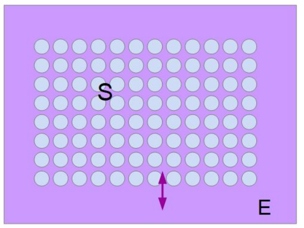

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Quantum Markov semigroup / Dissipative quantum evolution

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A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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The infinitesimal generator $\mathcal{L}_\Lambda$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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For $\rho_\Lambda \in \mathcal{S}_\Lambda$, $\mathcal{L}_\Lambda(\rho_\Lambda) = -i[H_\Lambda, \rho_\Lambda] + \sum_{k\in\Lambda} \widetilde{\mathcal{L}}_k(\rho_\Lambda)$ **GKLS equation**.

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We also assume that the quantum Markov process studied is **reversible**, i.e., it satisfies the **detailed balance condition** w.r.t. $\sigma \equiv \sigma_\Lambda$:

$$\langle f, \mathcal{L}_\Lambda^*(g) \rangle_\sigma = \langle \mathcal{L}_\Lambda^*(f), g \rangle_\sigma ,$$

for every $f, g \in \mathcal{B}_\Lambda$ and Hermitian, where

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Mixing time

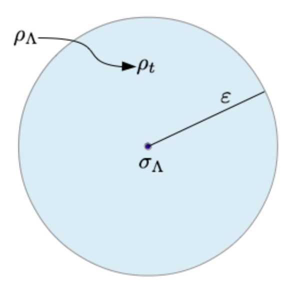
- Under the previous conditions, there is always convergence to σ_Λ.
- How fast does convergence happen?

Note $\mathcal{T}_\infty(\rho_\Lambda) := \sigma_\Lambda$ for every ρ_Λ.

Mixing time

We define the **mixing time** of $\{\mathcal{T}_t\}$ by

$$t_{\text{mix}}(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t(\rho_\Lambda) - \mathcal{T}_\infty(\rho_\Lambda) \right\|_1 \leq \varepsilon \right\}.$$



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We say that $\mathcal{L}_\Lambda$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

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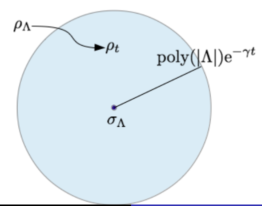

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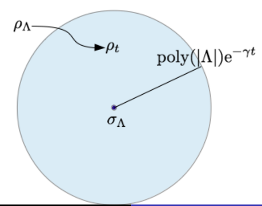

Applications to quantum information/quantum computing

What are the implications of rapid mixing?

Rapid mixing

$$\sup_{\rho \in \mathcal{S}(\mathcal{H}_\Lambda)} \|T_t(\rho) - \sigma\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}$$

Mixing time: $\tau(\varepsilon) = \mathcal{O}(\text{polylog}(|\Lambda|))$

“Negative” point of view:

- Quantum properties that hold in the ground state but not in the Gibbs state are **suppressed too fast** for them to be of any reasonable use.

“Positive” point of view:

- Thermal states with short mixing time can be **constructed efficiently** with a quantum device that simulates the effect of the thermal bath.
- This has important implications as a self-studying open problem as well as in optimization problems via simulated annealing type algorithms.

Applications to quantum information/quantum computing

If **rapid mixing**, no **error correction:**

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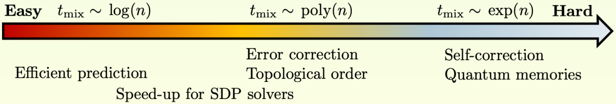

Main applications or consequences:

- Robust and efficient **preparation of topologically ordered phases** of matter via dissipation.
- Design of more efficient **quantum error-correcting codes** optimized for correlated Markovian noise models.
- **Stability** against local perturbations (Cubitt, Lucia, Michalakis, Pérez-García '15)
- **Area law** for mutual information (Brandao, Cubitt, Lucia, Michalakis, Pérez-García '15)
- Gaussian **concentration inequalities** (Lipschitz observables) (C., Rouzé, S. Franca '20)
- Finite blocklength refinement of **quantum Stein lemma** (C., Rouzé, Stilck Franca '20)
- **Quantum annealers:** Output an energy closed to that of the fixed point after short time (C., Rouzé, Stilck Franca '20)
- **Preparation Gibbs states:** Existence of local quantum circuits with logarithmic depth to prepare the Gibbs state (C., Rouzé, Stilck Franca '20)
- Establish the absence of **dissipative phase transitions** (Bardet, C., Gao, Lucia, Pérez-García, Rouzé '21)
- Examples of interacting **SPT phases** with decoherence time growing logarithmically with the system size for thermal noise (Bardet, C., Gao, Lucia, Pérez-García, Rouzé '21)

And many more. . .

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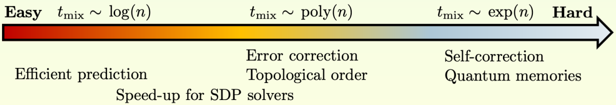

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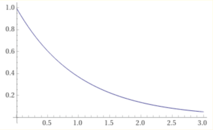

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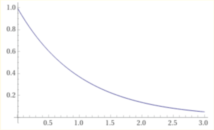

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If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) > 0$:

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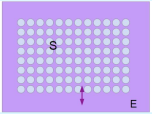

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Quantum spin systems

Kastoryano-Temme, '13

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$$D(T_t(\rho)\|\sigma) \leq D(\rho\|\sigma)\, \mathrm{e}^{-2\alpha(\mathcal{L})t}$$

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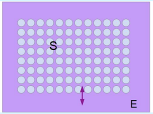

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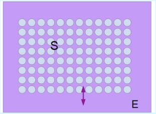

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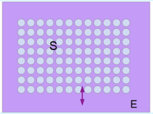

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Quantum spin systems

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KT'13

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Decay of correlations

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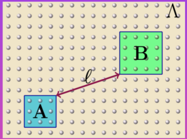

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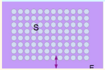

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Decay of correlations on Gibbs state

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial B}$,
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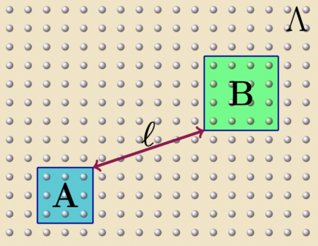

$\ell := \mathrm{dist}(A, B)$

Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta H_{A\cup B}} \approx \mathrm{e}^{-\beta H_A} \, \mathrm{e}^{-\beta H_B} \; ?$$

$$\mathrm{tr}_{A^c}[\sigma_\Lambda] \otimes \mathrm{tr}_{B^c}[\sigma_\Lambda] := (\sigma_\Lambda)_A \otimes (\sigma_\Lambda)_B \approx \mathrm{tr}_{(A\cup B)^c}[\sigma_\Lambda] := (\sigma_\Lambda)_{A\cup B} \; ?$$

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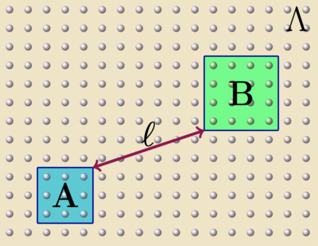

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Decay of correlations on Gibbs state

3 different forms of **decay of correlations**.

Operator correlation

$$\mathrm{Cov}_\sigma(A:B) := \sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]|$$

Mutual information

$$I_\sigma(A:B) := D(\sigma_{AB}||\sigma_A \otimes \sigma_B)$$

for $D(\rho||\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

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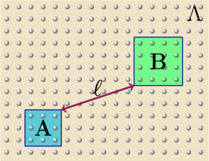
Mixing condition

$$\|h(\sigma_{AB})\|_\infty = \left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty$$



Relation:

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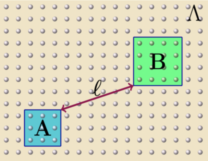

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Quantum spin systems

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Cubitt et al.'13 →

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Kastoryano-Brandao'14 →

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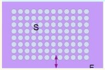

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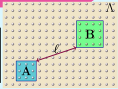

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KB'14

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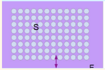

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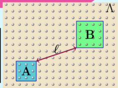

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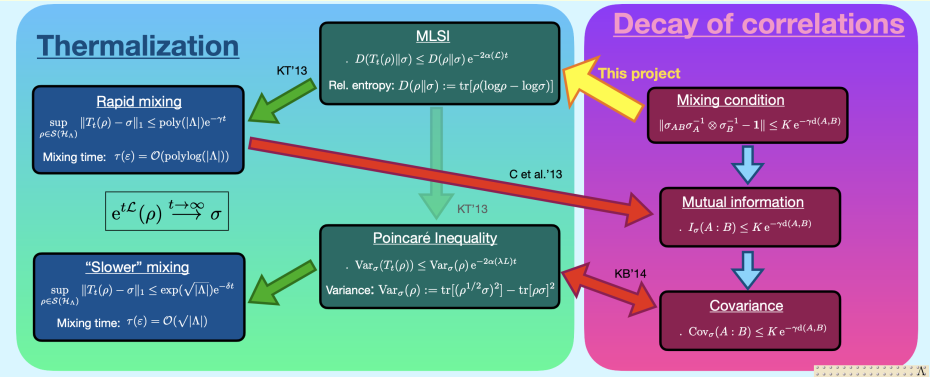

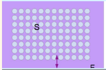

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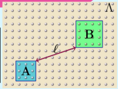

Setting and questions

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- H_Λ local (commuting) Hamiltonian $\quad \mapsto \quad \sigma_\Lambda := \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}[e^{-\beta H_\Lambda}]}$ Gibbs state .
- $\mathcal{L}_\Lambda$ local Lindbladian with unique stationary state σ_Λ ($\mathcal{L}_\Lambda(\sigma_\Lambda) = 0$).

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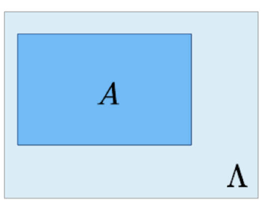
MLSI constant

$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) \geq \Psi(|\Lambda|) > 0 \qquad (\text{or } = 0 \text{ very "slowly", like } \Omega\left(\frac{1}{\text{poly} \log(|\Lambda|)}\right))$$



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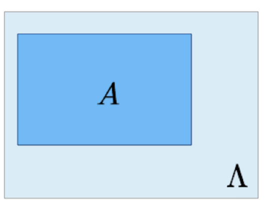

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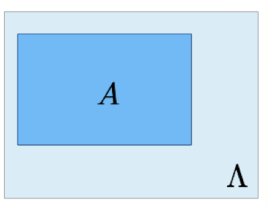

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No, but we can prove

$$\alpha(\mathcal{L}_\Lambda) \geq \Psi(|A|)\, \alpha_\Lambda(\mathcal{L}_A) > 0 \ .$$

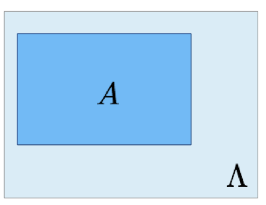
Objective

MLSI constant

$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) \geq \Psi(|\Lambda|) > 0 \qquad (\text{or } = 0 \text{ very “slowly”, like } \Omega\left(\frac{1}{\text{poly } \log(|\Lambda|)}\right))$$



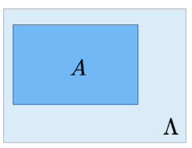
Can we prove something like

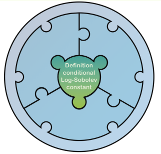
$$\alpha(\mathcal{L}_\Lambda) \geq \Psi(|A|)\, \alpha(\mathcal{L}_A) > 0 \ ?$$

No, but we can prove

$$\alpha(\mathcal{L}_\Lambda) \geq \Psi(|A|)\, \alpha_\Lambda(\mathcal{L}_A) > 0 \ .$$

Conditional MLSI constant





MLSI constant

The **MLSI constant** of $\mathcal{L}_\Lambda = \sum_{k \in \Lambda} \mathcal{L}_k$ is defined by

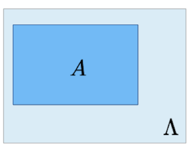
$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

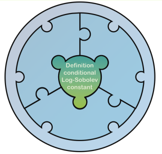
Conditional MLSI constant

The **conditional MLSI constant** of $\mathcal{L}_\Lambda$ on $A \subset \Lambda$ is defined by

$$\alpha_\Lambda(\mathcal{L}_A) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_A(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_A(\rho_\Lambda || \sigma_\Lambda)}$$

Conditional MLSI constant





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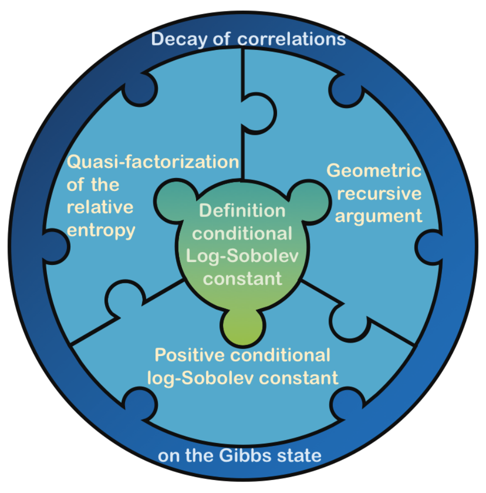
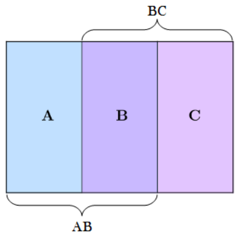
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

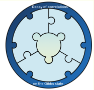
How does the strategy work?

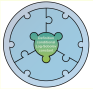
We want to prove:

$\alpha(\mathcal{L}_\Lambda) := \inf\limits_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda \| \sigma_\Lambda)}$

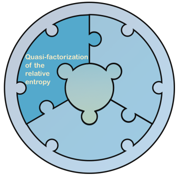
$$\boxed{\alpha(\mathcal{L}_\Lambda) \geq \Psi(|A|)\, \alpha_\Lambda(\mathcal{L}_A) > 0}$$

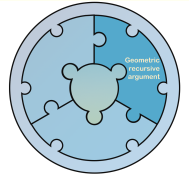
$\alpha_\Lambda(\mathcal{L}_A) := \inf\limits_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_A(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_A(\rho_\Lambda \| \sigma_\Lambda)}$

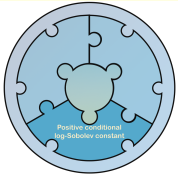
After choosing 

and 

, we prove the following:







$D(\rho_\Lambda \| \sigma_\Lambda) \to D_A(\rho_\Lambda \| \sigma_\Lambda)$

$\Psi(|A|) > 0$

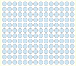
$\alpha_\Lambda(\mathcal{L}_A) > 0$

Example: Tensor product fixed point

(C.-Lucia-Pérez García '18)
(Beigi-Datta-Rouzé '18)

$$\mathcal{L}_\Lambda(\rho_\Lambda) = \sum_{x\in\Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$$ **heat-bath**

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

$$\leq \sum_{x\in\Lambda} D_x(\rho_\Lambda || \sigma_\Lambda)$$

$\alpha_\Lambda(\mathcal{L}_x^*) := \inf\limits_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 D_x(\rho_\Lambda || \sigma_\Lambda)}$

$$\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x)}$$

$$\leq \frac{1}{2 \inf\limits_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}_x)} \sum_{x\in\Lambda} - \operatorname{tr}[\mathcal{L}_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]$$

$$= \frac{1}{2 \inf\limits_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}_x)} \left(- \operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right)$$

$$\leq \left(- \operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right) .$$

Dynamics

Let $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}\left[e^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

$$\mathcal{L}_\Lambda^H(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right)$$

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Davies generator

The **Davies generator** is given by:

$$\mathcal{L}_\Lambda^{D;*}(X) := i[H_\Lambda, X] + \sum_{x \in \Lambda} \widetilde{\mathcal{L}}_x^D(X) \, ,$$

where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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The **Schmidt generator** (Bravyi-Vyalyi '05) can be written as:

$$\mathcal{L}_\Lambda^{S;*}(X) = \sum_{x\in\Lambda} \left(E_x^{S;*}(X) - X \right),$$

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Previous results

Let us recall: For $\alpha(\mathcal{L}_\Lambda)$ a MLSI constant,

$$\left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \sqrt{2 \log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda)\, t}.$$

Using the spectral gap $\lambda(\mathcal{L}_\Lambda)$:

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Spectral gap for Davies and heat-bath (Kastoryano-Brandao, '16)

Let $\mathcal{L}_\Lambda^{H,D}$ be the **heat-bath** or **Davies** generator in 1D. Then, $\mathcal{L}_\Lambda^{H,D}$ has a positive spectral gap that is independent of the system size, for every temperature.

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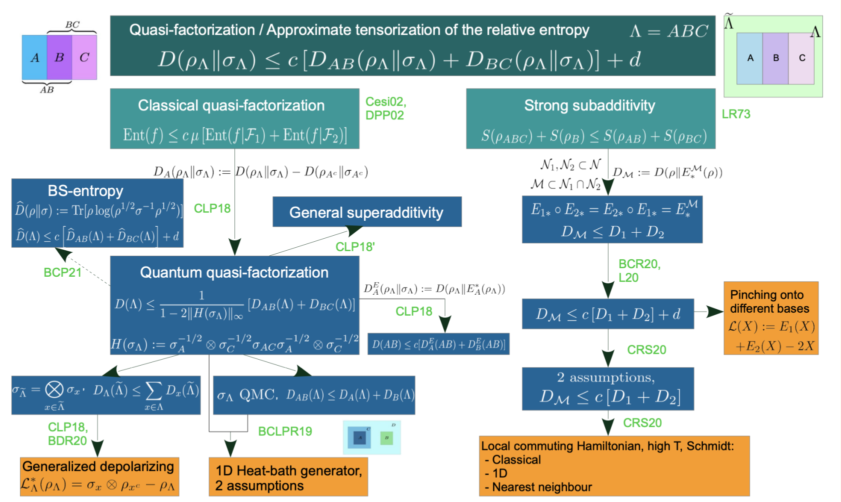
Quasi-factorization of the relative entropy

Results of **Quasi-Factorization** or **Approximate Tensorization**

Results of **Modified Logarithmic Sobolev Inequality**



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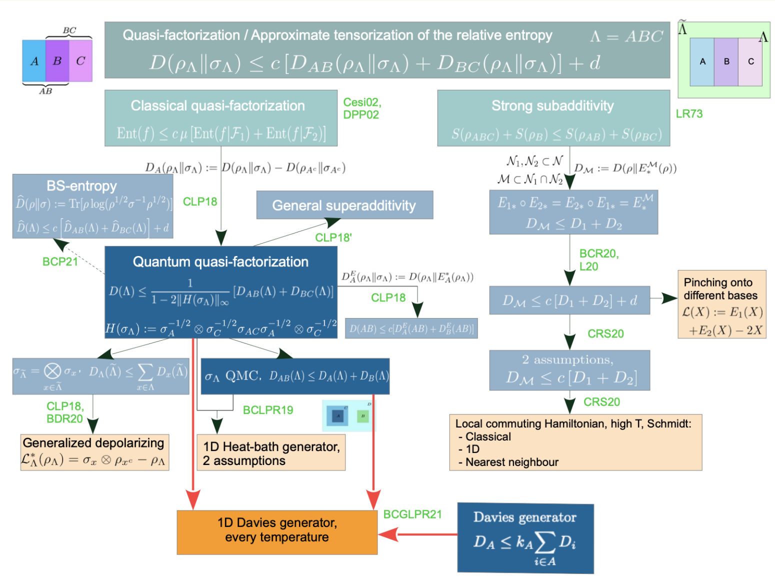

MLSI for Davies generators in 1D

MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '22)

Let $\mathcal{L}_\Lambda^D$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^D$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^D) = \Omega(\ln(|\Lambda|)^{-1})$.

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Rapid mixing:

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \mathrm{poly}(|\Lambda|) e^{-\gamma t}.$$

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In the setting above, $\mathcal{L}_\Lambda^D$ has rapid mixing.

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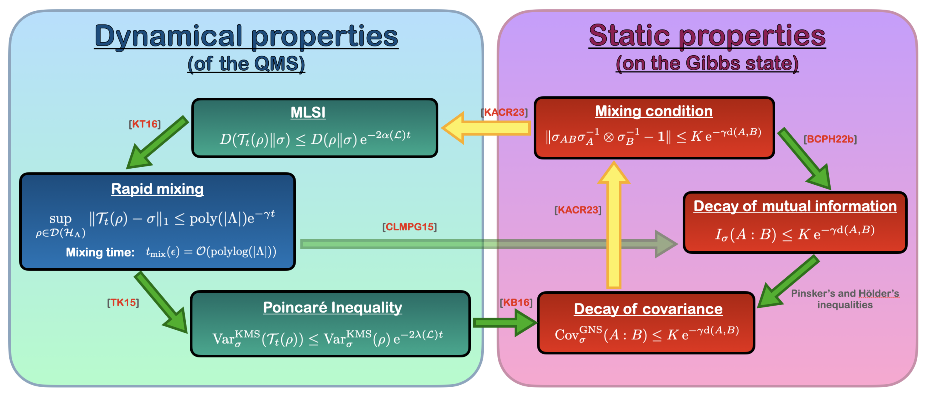
Main result


Dynamical properties
(of the QMS)
Static properties
(on the Gibbs state)
MLSI
$D(\mathcal{T}_t(\rho)\|\sigma) \leq D(\rho\|\sigma)\, e^{-2\alpha(\mathcal{L})t}$
[KT16]
[KACR23]
Mixing condition
$\|\sigma_{AB}\sigma_A^{-1}\otimes\sigma_B^{-1} - \mathbb{1}\| \leq K\, e^{-\gamma d(A,B)}$
[BCPH22b]
Rapid mixing
$\sup_{\rho\in\mathcal{D}(\mathcal{H}_\Lambda)} \|\mathcal{T}_t(\rho)-\sigma\|_1 \leq \text{poly}(|\Lambda|)e^{-\gamma t}$
Mixing time: $t_{\text{mix}}(\epsilon) = \mathcal{O}(\text{polylog}(|\Lambda|))$
[CLMPG15]
[KACR23]
Decay of mutual information
$I_\sigma(A:B) \leq K\, e^{-\gamma d(A,B)}$
[TK15]
Poincaré Inequality
$\text{Var}_\sigma^{\text{KMS}}(\mathcal{T}_t(\rho)) \leq \text{Var}_\sigma^{\text{KMS}}(\rho)\, e^{-2\lambda(\mathcal{L})t}$
[KB16]
Decay of covariance
$\text{Cov}_\sigma^{\text{GNS}}(A:B) \leq K\, e^{-\gamma d(A,B)}$
Pinsker's and Hölder's inequalities

MLSI for 2-colorable graphs

MLSI for 2-colorable graphs, (Alhambra-C.-Kochanowski-Rouzé, '23)

Let Λ be a 2-colorable graph and $\mathcal{L}_\Lambda^D$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, 2-local Hamiltonian. If:

i) The Lindbladian is **gapped**.

ii) The Gibbs state satisfies **exponential decay of covariance**.

Then, $\mathcal{L}_\Lambda^D$ satisfies a **MLSI** with constant

1) $\alpha(\mathcal{L}_\Lambda^D) = \Omega(1)_{|\Lambda|\to\infty}$, when Λ is a sub-exponential graph (e.g. hypercubic lattice), or

2) $\alpha(\mathcal{L}_\Lambda^D) = \Omega\left((\ln|\Lambda|)^{-1}\right)_{|\Lambda|\to\infty}$, if the correlation length of the fixed point is sufficiently small (e.g. b-ary trees).

Rapid mixing

i) $\Lambda = \mathbb{Z}$ is 1-dimensional, H_Λ is k-local and $\beta > 0 \Rightarrow \mathcal{L}_\Lambda^D$ has a constant MLSI.

ii) $\Lambda = \mathbb{Z}^D$ is D-dimensional, H_Λ is 2-local and $\beta < \beta_* \Rightarrow \mathcal{L}_\Lambda^D$ has a constant MLSI.

iii) $\Lambda = \mathbb{T}_b$ is an inf. b-ary tree, H_Λ is 2-local and $\beta < \beta_* \Rightarrow \mathcal{L}_\Lambda^D$ has a log-size MLSI.

In all cases, $\mathcal{L}_\Lambda^D$ satisfies **rapid mixing**.

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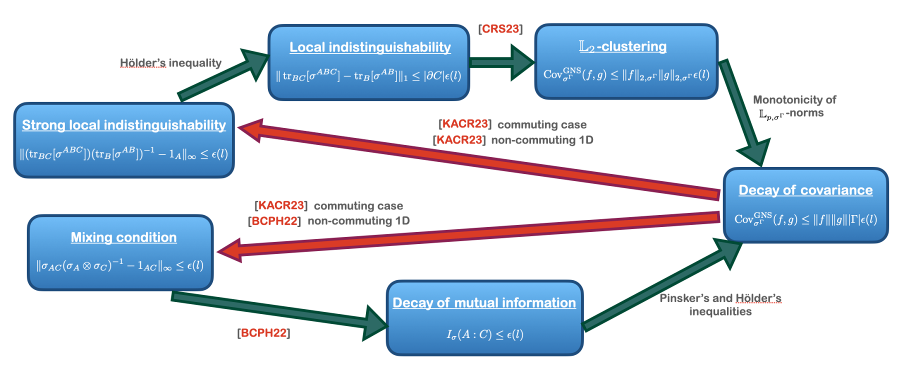
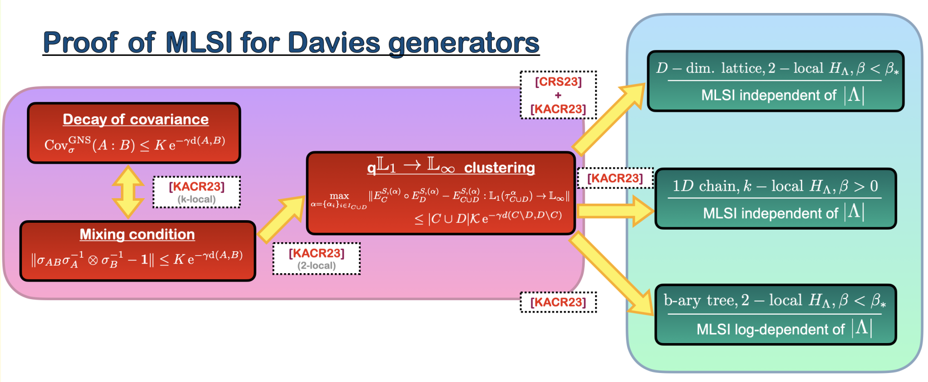
In all cases, $\mathcal{L}_\Lambda^D$ satisfies **rapid mixing**.

Ingredients of the proof



For k-local, commuting Hamiltonians, **exponential decay of covariance** implies **mixing condition**.

Ingredients of the proof



The last part uses "divide and conquer" arguments for the relative entropy.

$$+$$

Equivalence between dynamics:

$$D(\rho \| E_X^D(\rho)) \leq D(\rho \| E_X^S(\rho)) \leq D(\rho \| E_{X\partial}^D(\rho))$$

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