Exponential decay of mutual information for Gibbs states of local Hamiltonians

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Joint work with: **Andreas Bluhm** (U. Copenhagen)
Antonio Pérez-Hernández (UNED, Spain) .

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Overview

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

3 different forms of **decay of correlations**.

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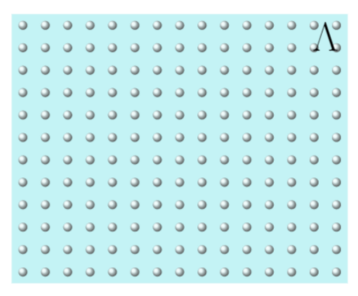

- A finite lattice $\Lambda \subset \mathbb{Z}^D$.
- For each site $x \in \Lambda$, $\mathcal{H}_x \equiv \mathbb{C}^d$.
- $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x \equiv (\mathbb{C}^d)^{\otimes |\Lambda|}$.
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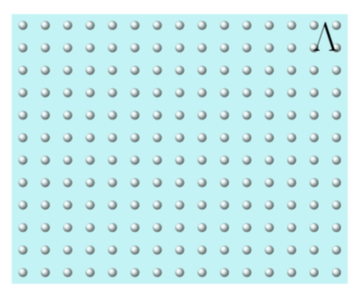

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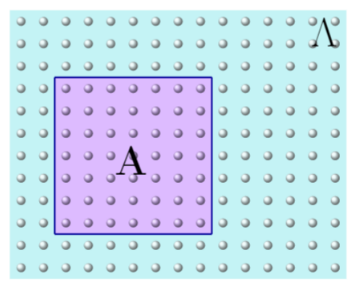

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Hamiltonian

- $H_\Lambda = \sum_{X \subset \Lambda} H_X$, with $\| H_X \| \leq J$ and $\| H_X \| = 0$ if $\operatorname{diam}(X) > r$.
- $\rho^\Lambda = \rho^\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} / \operatorname{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.

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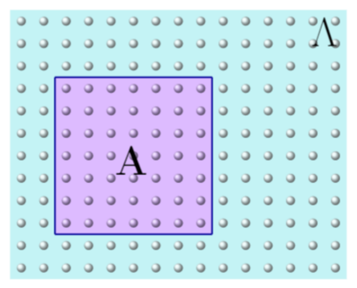

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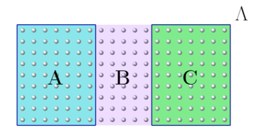

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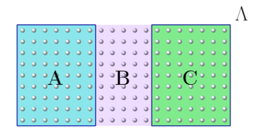

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Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D	Exp. (~ Araki, '69)	
Dimension	Large D		

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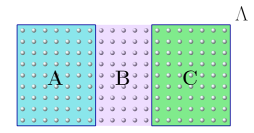

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Dimension	Large D		(same cell as 1 D)

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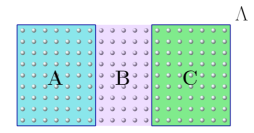

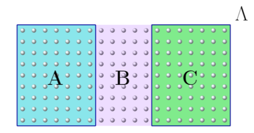

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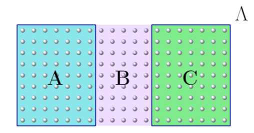

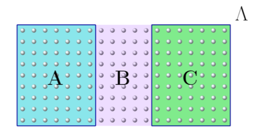

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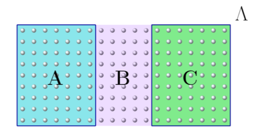

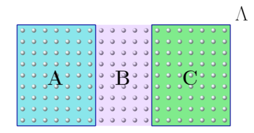

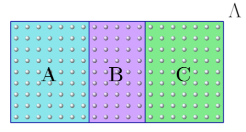

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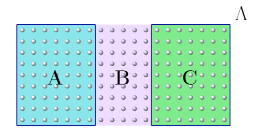

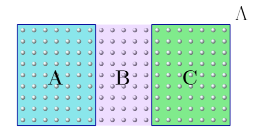

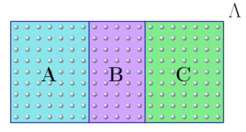

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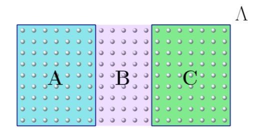

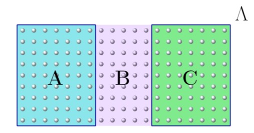

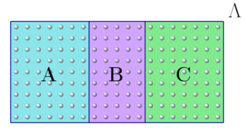

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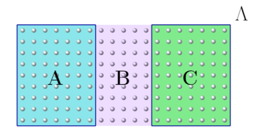

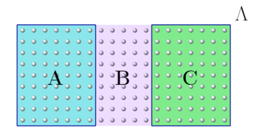

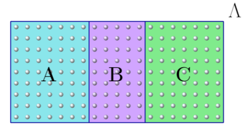

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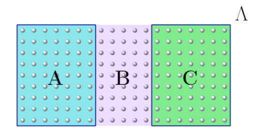

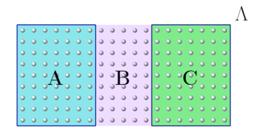

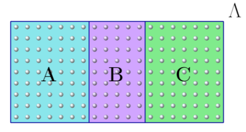

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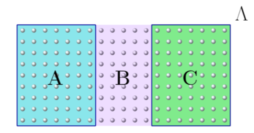

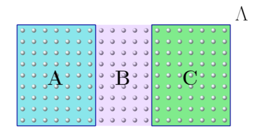

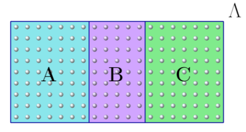

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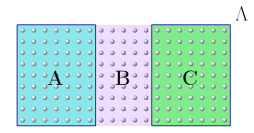
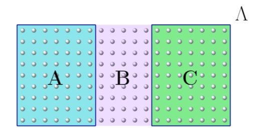
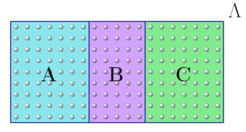
Conditional Mutual Information		Temperature	
		Low T	High T
Dimension	1 D	Subexp. (KB, '19)	Exp. (Kuwahara et al. '20)
	Large D		

Overview







Operator correlation

$$\Psi_X(Q) := \mathrm{Tr}[\rho^X Q]\,,\; Q \in \mathfrak{A}_X$$

$$\mathrm{Corr}_{\rho^\Lambda}(A:C) := \sup \left| \Psi_\Lambda(O_A O_C) - \Psi_\Lambda(O_A)\Psi_\Lambda(O_C) \right|$$

over $\|O_A\|, \|O_C\| \leq 1$.

Mutual information

$$I_\rho(A:C) := D(\rho_{AC} || \rho_A \otimes \rho_C)$$

for $D(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$

Conditional mutual inf.

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\mathrm{Tr}[\rho\log\rho]$

$$I_\rho(A:C) \geq \frac{1}{2}\,\mathrm{Corr}_\rho(A:C)^2$$

Decay:
$\mathrm{Corr}_{\rho^\Lambda}(A:C) \leq f(\mathrm{d}(A:C))$

Operator Correlation		Temperature: Low T	Temperature: High T
Dimension	1 D	Exponential	Exponential
Dimension	Large D		Exp. (Kliesch et al. '14)

Decay:
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Mutual Information		Temperature: Low T	Temperature: High T
Dimension	1 D	Exponential	Exponential
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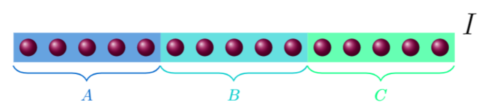
Exponential decay of correlations



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$$\Psi_I(Q) := \mathrm{Tr}[\rho^I Q]\,,\ Q \in \mathfrak{A}_I \quad \text{with } \rho^I = \mathrm{e}^{-\beta H_I} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$$

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Exponential uniform clustering

There exist $\mathcal{K}, \gamma > 0$ such that for every finite lattice $\Lambda = ABC$,

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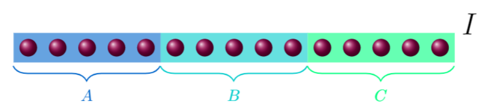

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Conjecture:

In **1D**, there are no thermal phase transitions.

$\Downarrow$

Exponential uniform clustering holds in **1D** at every $\beta > 0$.

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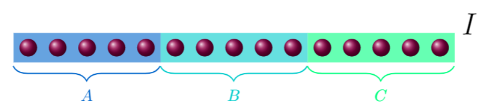

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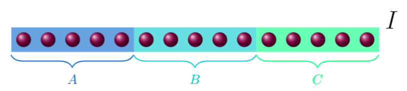

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There exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$,

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Theorem (Araki, '69)

- $\mathfrak{A}_\mathbb{Z}$ algebra of quasi-local observables.
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Assume translation-invariant interactions. There exist $\mathcal{K}', \gamma' > 0$ such that for every finite interval $I = ABC$,

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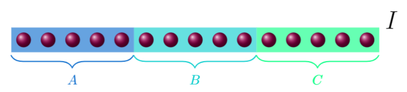

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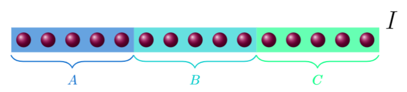

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$\Downarrow$ (easy) $\Uparrow$ **Thm.** (Bluhm-C.-Perez Hernandez '21)

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Decay of mutual information

Mutual information

$$I_\rho(A:B) := D(\rho_{AB}||\rho_A \otimes \rho_B)\,.$$

By Hölder and Pinsker's inequalities:

$$I_\rho(A:B) \geq \frac{1}{2}\|\rho_{AB} - \rho_A \otimes \rho_B\|_1^2 \geq \frac{1}{2}\,\mathrm{Corr}_\rho(A:B)^2\,.$$

Decay of mutual information $\overset{\Rightarrow}{\not\Leftarrow}$ Decay of correlations

There are states with small operator correlation and large mutual information in quantum data hiding (Hayden et al. '04).

Area laws

$$I_\rho(A:A^c) \leq O(|\partial A|)\,.$$

Wolf et al., '08: Area laws for the mutual information (improved: Kuwahara et al. '20).

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$$\text{Decay of mutual information} \; \begin{matrix} \Rightarrow \\ \not\Leftarrow \end{matrix} \; \text{Decay of correlations}$$

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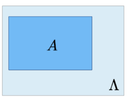

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BS-entropy and geometric Rényi divergences

Belavkin-Staszewski relative entropy

$$\widehat{D}(\rho_{AC} \,||\, \sigma_{AC}) := \operatorname{Tr}\left[\rho_{AC} \log \left(\rho_{AC}^{1/2} \sigma_{AC}^{-1} \rho_{AC}^{1/2} \right) \right] .$$

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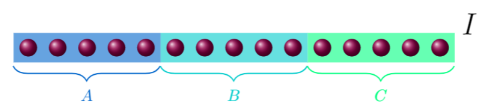
Exponential decay of mutual information in 1D



Exponential decay mixing condition (Bluhm-C.-Pérez Hernández, '21)

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I}/\mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

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Corollary (Bluhm-C.-Pérez Hernández, '21)

The following are equivalent:

1. Exponential decay of correlations for the infinite-chain thermal states.
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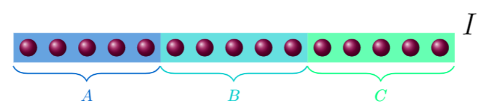

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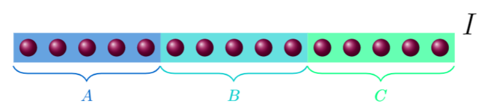

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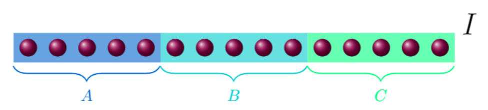
Sketch of the proof



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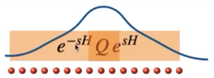
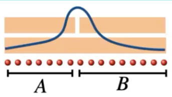
Ingredients:



- Imaginary-time **Lieb-Robinson bounds** (Araki, '69)



- Araki's **expansionals** (Araki, '69): $E = \mathrm{e}^{H_A + H_B}\mathrm{e}^{-H_{AB}}$
- **Exponential uniform clustering**
- **Local indistinguishability** (Brandao-Kastoryano, '19)

Conditional Mutual Information

Conditional mutual information

$$I_\rho(A:C|B) := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC})$$

for $S(\rho) = -\text{Tr}[\rho \log \rho]$.

Strong subadditivity and recoverability (Lieb-Ruskai, '73, Petz, '86)

$$I_\rho(A:C|B) \geq 0$$

and $I_\rho(A:C|B) = 0$ if, and only if, ρ_{ABC} is a quantum Markov chain $(A \leftrightarrow B \leftrightarrow C)$:

- $\rho_{ABC} = \rho_{AB}^{1/2} \rho_B^{-1/2} \rho_{BC} \rho_B^{-1/2} \rho_{AB}^{1/2}$.
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  $$\rho_{ABC} = \mathcal{R}_{B\to AB}(\rho_{BC}) \, .$$

In general, for any quantum channel $\mathcal{T}$ and positive states ρ and σ,

$D(\rho\|\sigma) \geq D(\mathcal{T}(\rho)\|\mathcal{T}(\sigma))$ and $D(\rho\|\sigma) = D(\mathcal{T}(\rho)\|\mathcal{T}(\sigma)) \;\Leftrightarrow\; \rho = \sigma^{\frac{1}{2}} \mathcal{T}^* \left(\mathcal{T}(\sigma)^{-\frac{1}{2}} \mathcal{T}(\rho) \mathcal{T}(\sigma)^{-\frac{1}{2}} \right) \sigma^{\frac{1}{2}}$

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Decay CMI in 1D (Kato-Brandao, '19)

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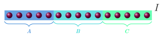

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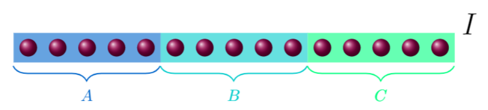
Approximate factorization of Gibbs states



BS - Conditional mutual information

$$\widehat{I}_\rho(A:C|B) := \widehat{D}(\rho_{ABC}||\rho_{AB}) - \widehat{D}(\rho_{BC}||\rho_B)$$

for $\widehat{D}(\rho||\sigma) = \mathrm{Tr}[\rho \log(\rho^{1/2}\sigma^{-1}\rho^{1/2})]$.

Bluhm-C., '20: For any quantum channel $\mathcal{T}$ and any positive states ρ and σ:

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For two positive states $\rho_{ABC}, \sigma_{ABC} \in \mathcal{H}_{ABC}$ such that $\sigma_{ABC} = \rho_{AB} \otimes \mathbb{1}_C/d_C$ and a $\mathcal{T} := \mathbb{1}_A/d_A \otimes \mathrm{Tr}_A$,

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Approximate factorization (Bluhm-C.-Pérez Hernández, '21)

There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

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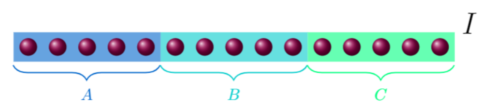

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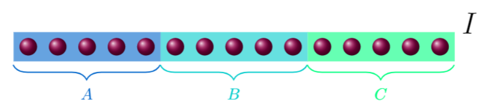
Applications to MLSI



Exponential decay mixing condition

Assuming exponential uniform clustering, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I} / \mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

$$\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \| \leq \mathcal{K} \, \mathrm{e}^{-\gamma |B|}$$

MLSI for heat-bath dynamics (Bardet-C.-Lucia-Perez Garcia-Rouze, '21)

Theorem 7. *Let $\Lambda \subset\subset \mathbb{Z}$ be a finite chain. Let $\Phi : \Lambda \to \mathcal{A}_\Lambda$ be a k-local commuting potential and $H_\Lambda = \sum_{x \in \Lambda} \Phi(x)$ be its corresponding Hamiltonian, and denote by σ_Λ its Gibbs state. Let $\mathcal{L}_\Lambda^*$ be the generator of the heat-bath dynamics. Then, if Assumptions 1 and 2 hold, the MLSI constant of $\mathcal{L}_\Lambda^*$ is strictly positive and independent of $|\Lambda|$.*

Assumption 1 (mixing condition). *Let $\Lambda \subset\subset \mathbb{Z}$ be a finite chain, and let $C, D \subset \Lambda$ be the union of non-overlapping finite-sized segments of Λ. Let σ_Λ be the Gibbs state of a commuting Hamiltonian. The following inequality holds for certain positive constants K_1, K_2 independent of Λ, C, D:*

$$\left\| \sigma_C^{-1/2} \otimes \sigma_D^{-1/2} \sigma_{CD} \, \sigma_C^{-1/2} \otimes \sigma_D^{-1/2} - \mathbb{1}_{CD} \right\|_\infty \leq K_1 \mathrm{e}^{-K_2 d(C,D)},$$

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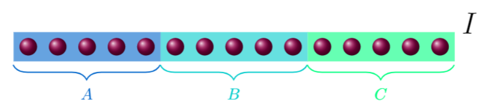
Applications to MLSI



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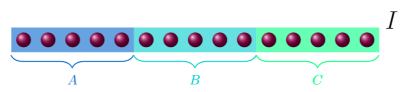

Exponential decay mixing condition

For translation-invariant interactions, there exist $\mathcal{K}, \gamma > 0$ such that for every finite interval $I = ABC$ and $\rho = \rho^I = \mathrm{e}^{-\beta H_I}/\mathrm{Tr}[\mathrm{e}^{-\beta H_I}]$,

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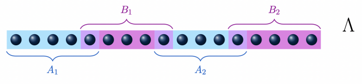

MLSI for heat-bath dynamics (Bardet-C.-Gao-Lucia-Perez Garcia-Rouze, '21)

Positive MLSI ($O(\log(|I|))$) for Davies generators in 1D at any $\beta > 0$.

Theorem 3.1. *Let $\Lambda = [\![1, n]\!]$. For any $\beta > 0$, we denote by $\sigma \equiv \sigma^\beta$ the Gibbs state of a finite-range, translation-invariant, commuting Hamiltonian at inverse temperature $\beta > 0$. Consider $\mathcal{L}^D_{\Lambda *}$ the Davies generator of a quantum Markov semigroup $\{e^{t\mathcal{L}^D_{\Lambda *}}\}_{t \geq 0}$ with unique fixed point σ. Then, there exists $\alpha_n = \Omega(\ln(n)^{-1})$ such that, for all $\rho \in \mathcal{D}(\mathcal{H}_\Lambda)$ and all $t \geq 0$,*

$$D(\rho_t \| \sigma) \leq e^{-\alpha_n t} D(\rho \| \sigma) \,, \tag{25}$$

where $\rho_t := e^{t\mathcal{L}^D_{\Lambda *}}(\rho)$. Moreover, $\alpha_n = e^{-\mathcal{O}(\beta)}$ as a function of β.

Applications to MPO approximation

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There exists a positive function $\ell \longmapsto \delta(\ell)$, exhibiting **superexponential** decay, such that for every three adjacent and finite intervals ABC,

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Approximation by MPOs (Bluhm-C.-Pérez Hernández, '22)

There exists $\mathcal{G}, \mathcal{K} > 1$ such that for every finite interval $I \subset \mathbb{Z}$, split into $I = A_1 A_2 \ldots A_n$ with $|A_j| = m$ for every $j = 1, \ldots, n$, and for $\rho = \rho^I$ the Gibbs state on I,

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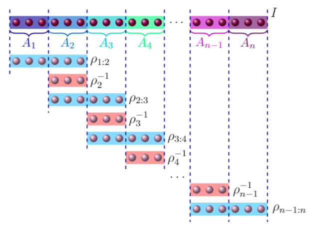

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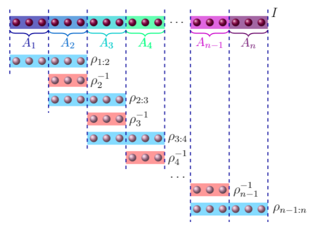

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