Logarithmic Sobolev Inequalities for Quantum Many-Body Systems

Ángela Capel (Technische Universität München)

Joint work with: **Ivan Bardet** (INRIA, Paris),
Angelo Lucia (Caltech),
Cambyse Rouzé (T. U. München) and
David Pérez-García (U. Complutense de Madrid).

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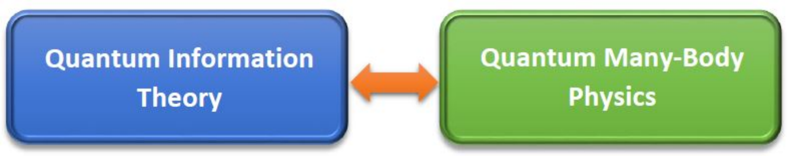
Technical University of Munich

Munich Center for Quantum Science and Technology

Quantum



Communication channels $\longleftrightarrow$ Physical interactions

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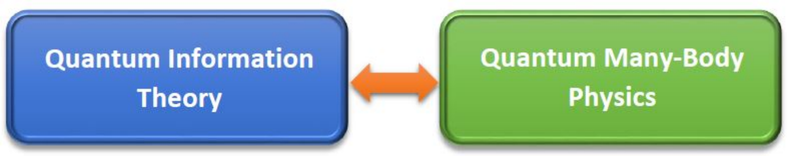

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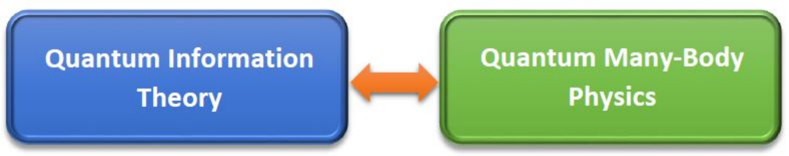

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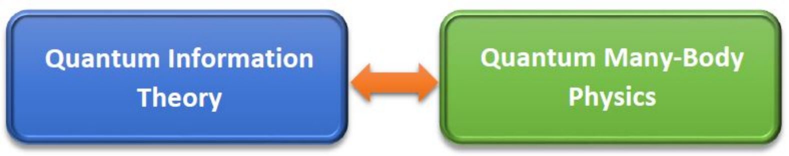

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Field of study

Dissipative evolutions of quantum many-body systems

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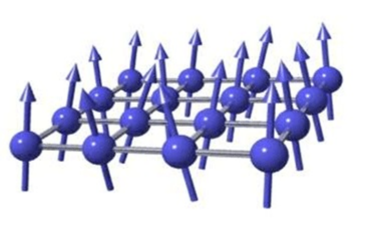
Contents

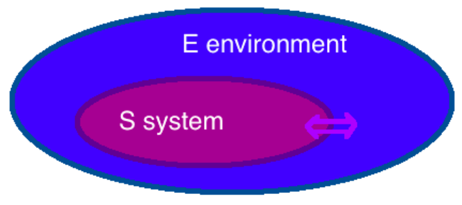
1. Introduction and motivation



Open quantum systems

No experiment can be executed at zero temperature or be completely shielded from noise.

$\Rightarrow$ Open quantum many-body systems.



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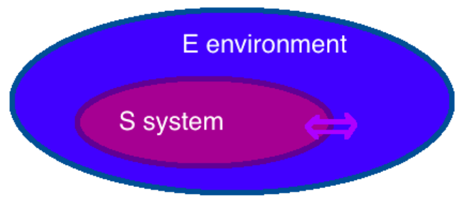

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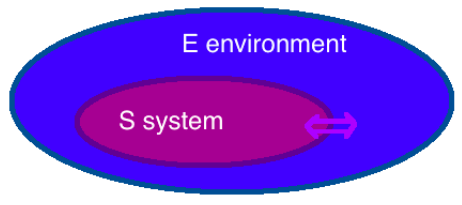

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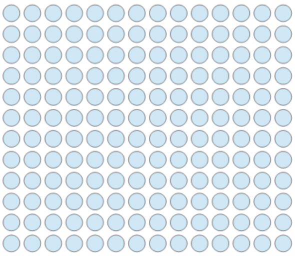
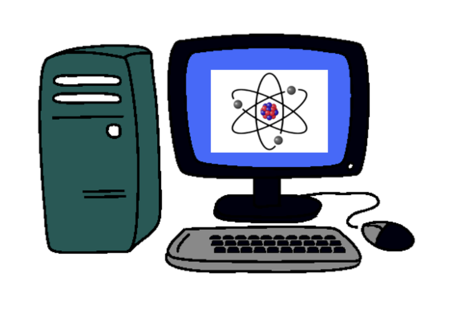


Figure: A quantum spin lattice system.

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- To every site $x \in \Lambda$ we associate $\mathcal{H}_x$ $(= \mathbb{C}^D)$.
- The global Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$.
- The set of bounded linear endomorphisms on $\mathcal{H}_\Lambda$ is denoted by $\mathcal{B}_\Lambda := \mathcal{B}(\mathcal{H}_\Lambda)$.
- The set of density matrices is denoted by $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \ : \ \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

Quantum dissipative evolutions useful?

Main motivation:



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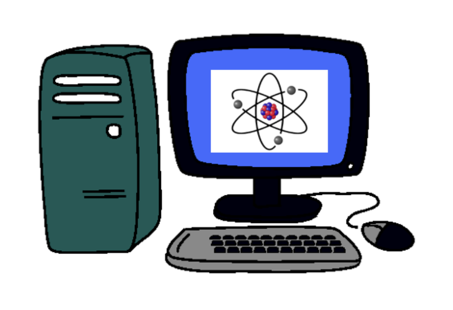

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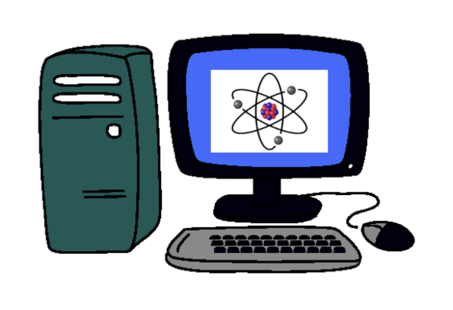

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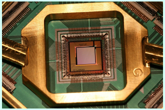

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New area:

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to create artificial evolutions in which the dissipative process works in favor (protecting the system from noisy evolutions).

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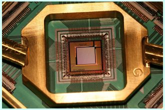

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- Conditions against noise
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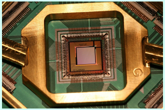

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Physical evolution: $\rho \mapsto U \rho U^*$ $\rightsquigarrow$ Reversible

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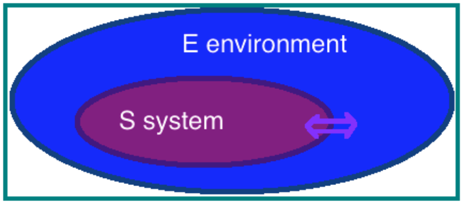

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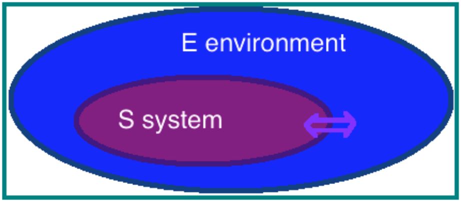

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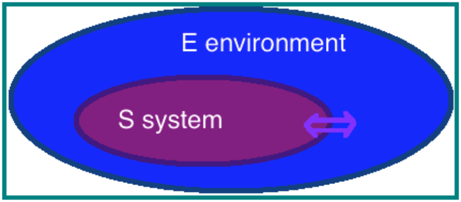

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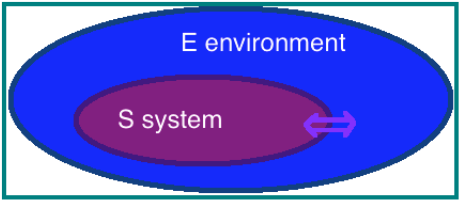

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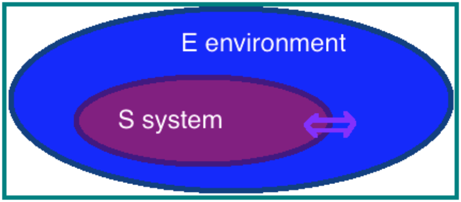

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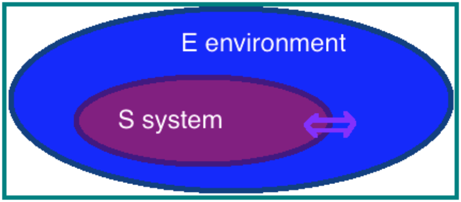

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A **quantum dissipative system** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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Primitive QMS

We assume that $\{\mathcal{T}_t^*\}_{t\geq 0}$ has a unique full-rank invariant state, which we denote by σ.

Reversibility

We also assume that the quantum Markov process studied is **reversible**, i.e. it satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}(g)\rangle_\sigma = \langle \mathcal{L}(f), g\rangle_\sigma$$

for every $f, g \in \mathcal{A}$, in the Heisenberg picture.

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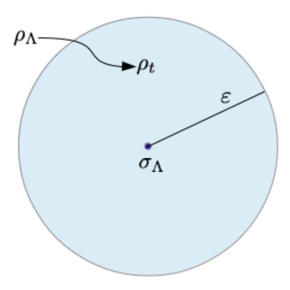
Mixing time

We define the **mixing time** of $\{\mathcal{T}_t^*\}$ by

$$\tau(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t^*(\rho) - \mathcal{T}_\infty^*(\rho) \right\|_1 \leq \varepsilon \right\}.$$



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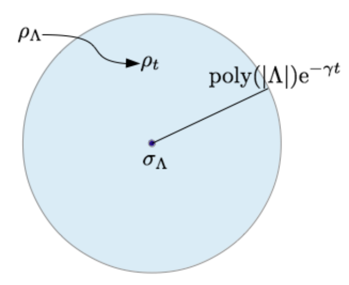

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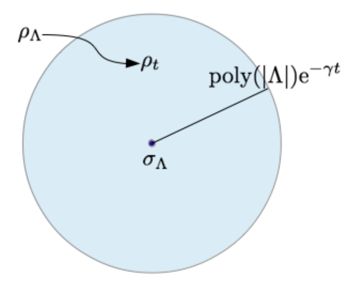

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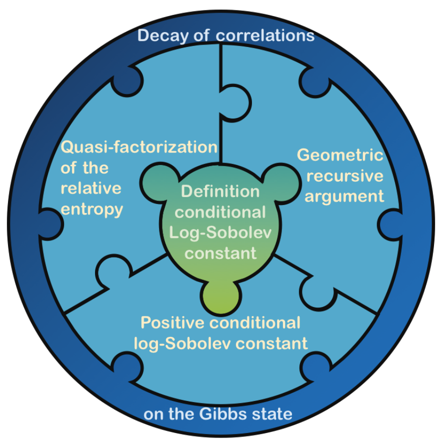
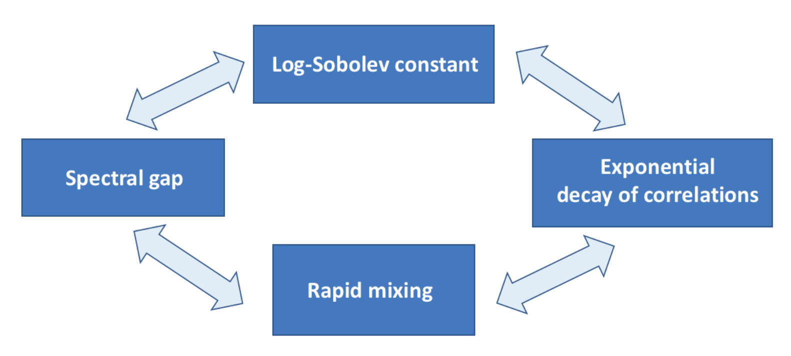
2. General strategy for log-Sobolev inequalities



Classical spin systems



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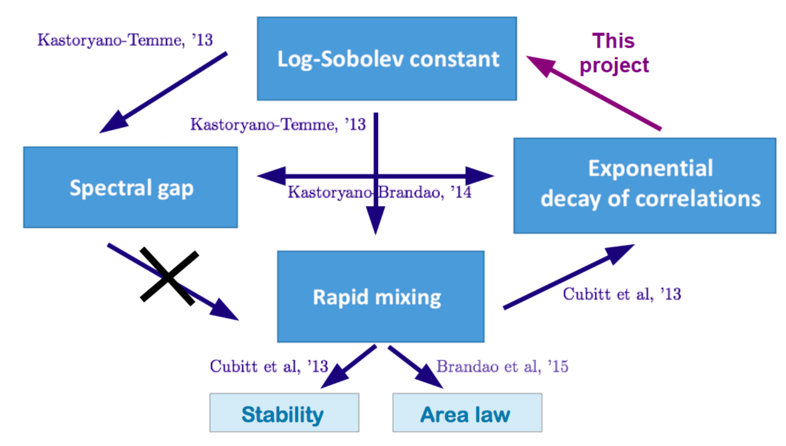

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Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

Liouville's equation:

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$$D(\rho_t||\sigma_\Lambda) = \operatorname{tr}[\rho_t(\log \rho_t - \log \sigma_\Lambda)].$$

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Lower bound for the derivative of $D(\rho_t || \sigma_\Lambda)$ in terms of itself:

$$2\alpha D(\rho_t || \sigma_\Lambda) \leq -\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)].$$

Log-Sobolev inequality (MLSI)

Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

Liouville's equation:

$$\partial_t \rho_t = \mathcal{L}_\Lambda^*(\rho_t).$$

Relative entropy of ρ_t and σ_Λ:

$$D(\rho_t||\sigma_\Lambda) = \operatorname{tr}[\rho_t(\log \rho_t - \log \sigma_\Lambda)].$$

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$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) > 0$:

$$D(\rho_t || \sigma_\Lambda) \leq D(\rho_\Lambda || \sigma_\Lambda) e^{-2\,\alpha(\mathcal{L}_\Lambda^*)\,t},$$

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and with **Pinsker's inequality**, we have:

$$\|\rho_t - \sigma_\Lambda\|_1 \leq \sqrt{2D(\rho_\Lambda||\sigma_\Lambda)}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t} \leq \sqrt{2\log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t}.$$

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Log-Sobolev constant

Rapid mixing

We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|)\, e^{-\gamma t}.$$

For thermal states, $\sigma_{\min}^{-1} \sim \exp(|\Lambda|)$.

Log-Sobolev constant

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Find positive log-Sobolev constants!

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First main objective of this talk

Develop a strategy to find positive log-Sobolev constants from static properties on the fixed point.

Second main objective of this talk

Apply that strategy to certain dissipative dynamics.

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Classical spin systems

(Cesi, Dai Pra-Paganoni-Posta, '02)

(1) Quasi-factorization of the entropy (in terms of a conditional entropy).

+

(2) Recursive geometric argument.
Lower bound for the global log-Sobolev constant in terms of the log-Sobolev constant of a size-fixed region.

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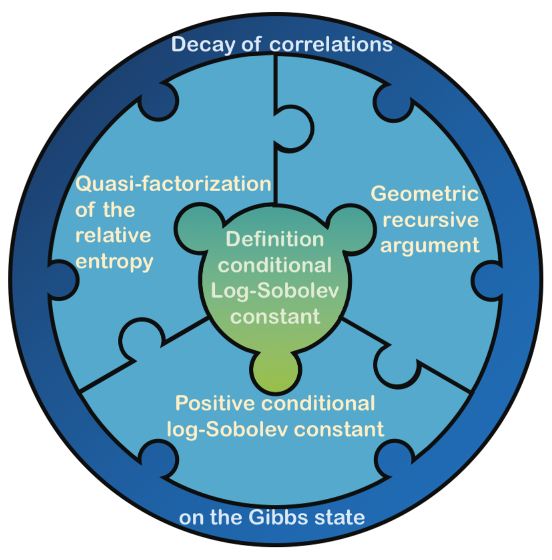
Strategy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Conditional log-Sobolev constant

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Let $\mathcal{L}_\Lambda^* : \mathcal{S}_\Lambda \to \mathcal{S}_\Lambda$ be a primitive reversible Lindbladian with stationary state σ_Λ. We define the **log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ by

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Conditional log-Sobolev constant

Let $\mathcal{L}_\Lambda^* : \mathcal{S}_\Lambda \to \mathcal{S}_\Lambda$ be a primitive reversible Lindbladian with stationary state σ_Λ, $A \subseteq \Lambda$. We define the **conditional log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ on A by

$$\alpha_\Lambda(\mathcal{L}_A^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_A^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_A(\rho_\Lambda || \sigma_\Lambda)}$$

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Conditional relative entropy

Classical entropy and conditional entropy

Entropy:

$$\mathrm{Ent}_\mu(f) = \mu(f \log f) - \mu(f) \log \mu(f).$$

Conditional entropy:

$$\mathrm{Ent}_\mu(f \mid \mathcal{G}) = \mu(f(\log f - \log \mu(f \mid \mathcal{G})) \mid \mathcal{G}).$$

Quantum relative entropy

The **quantum relative entropy** of ρ_Λ and σ_Λ is defined by:

$$D(\rho_\Lambda || \sigma_\Lambda) = \mathrm{tr}\left[\rho_\Lambda(\log \rho_\Lambda - \log \sigma_\Lambda)\right].$$

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Given a bipartite space $\mathcal{H}_{AB}$, we define the conditional relative entropy in A by:

$$D_A(\rho_{AB}||\sigma_{AB}) = D(\rho_{AB}||\sigma_{AB}) - D(\rho_B||\sigma_B)$$

for every $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$.

(C.-Lucia-Pérez García, '18) $\rightarrow$ Axiomatic characterization of the CRE.

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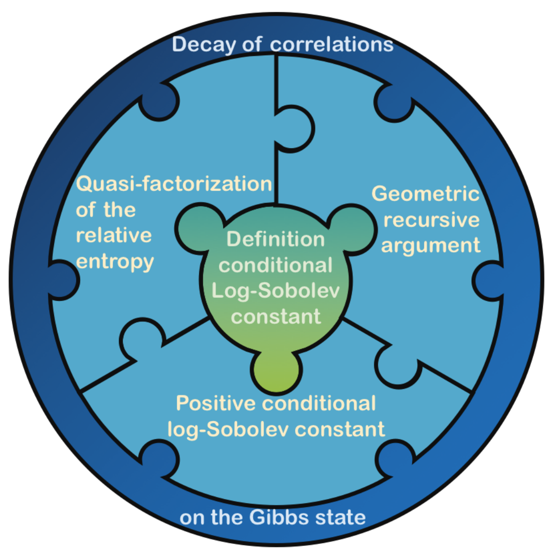
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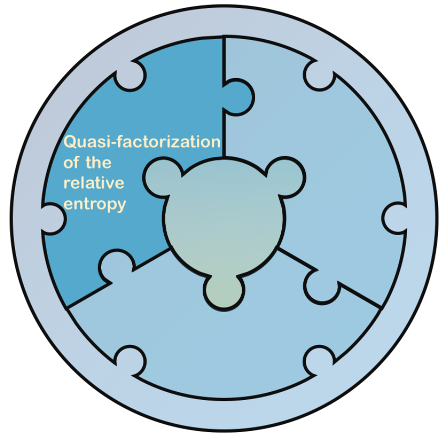
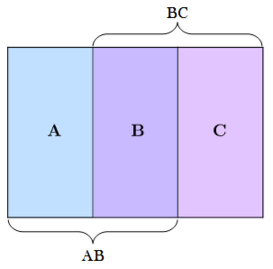
3. Quasi-factorization for the relative entropy



Quasi-factorization of the relative entropy

The strategy is based on a solution for the following problem.



Problem

Let $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ and $\rho_{ABC}, \sigma_{ABC} \in S_{ABC}$. Can we prove something like

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right]$$

where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$?

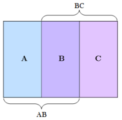


Figure: Choice of indices in $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$.

Result of **quasi-factorization** of the relative entropy, for every $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$:

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right].$$

Quasi-factorization for the CRE, (C.-Lucia-Pérez García, '18)

In the previous inequality,

$$\xi(\sigma_{ABC}) = \frac{1}{1 - 2\|H(\sigma_{AC})\|_\infty},$$

where

$$H(\sigma_{AC}) = \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \, \sigma_{AC} \, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC}.$$

Note that $H(\sigma_{AC}) = 0$ if σ_{AC} is a tensor product between A and C.

$$
\begin{aligned}
(1 - 2\|H(\sigma_{AC})\|_\infty) D(\rho_{ABC}||\sigma_{ABC}) \leq \\
D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) = \\
= 2D(\rho_{ABC}||\sigma_{ABC}) - D(\rho_C||\sigma_C) - D(\rho_A||\sigma_A).
\end{aligned}
$$

$$\Leftrightarrow$$

$$(1 + 2\|H(\sigma_{AC})\|_\infty) D(\rho_{ABC}||\sigma_{ABC}) \geq D(\rho_A||\sigma_A) + D(\rho_C||\sigma_C).$$

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This result is equivalent to **(C.-Lucia-Pérez García, '18)**:

$$\boxed{(1+2\|H(\sigma_{AB})\|_\infty) D(\rho_{AB}||\sigma_{AB}) \geq D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)}.$$

Recall:

- **Superadditivity.** $D(\rho_{AB}||\sigma_A \otimes \sigma_B) \geq D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)$.

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Due to:

- **Monotonicity.** $D(\rho_{AB}||\sigma_{AB}) \geq D(T(\rho_{AB})||T(\sigma_{AB}))$ for every quantum channel T.

we have

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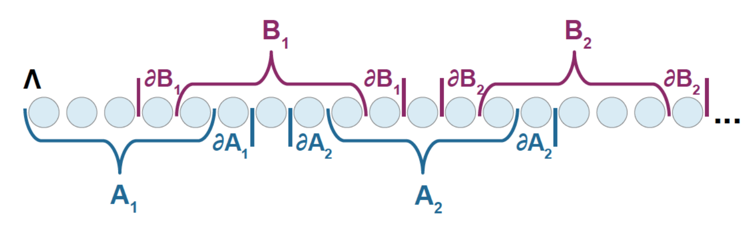
4. Logarithmic Sobolev inequalities



Example 1 (C.-Lucia-Pérez García, '18)

Heat-bath dynamics with tensor product fixed point

Heat-bath with tensor product fixed point

Theorem (C.-Lucia-Pérez García, '18)

The **heat-bath dynamics**, with tensor product fixed point, has a positive log-Sobolev constant.

Consider the local and global Lindbladians

$$\mathcal{L}_x^* := \mathbb{E}_x^* - \mathbb{1}_\Lambda, \; \mathcal{L}_\Lambda^* = \sum_{x \in \Lambda} \mathcal{L}_x^*$$

Since

$$\mathbb{E}_x^*(\rho_\Lambda) = \sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} = \sigma_x \otimes \rho_{x^c}$$

for every $\rho_\Lambda \in \mathcal{S}_\Lambda$, we have

$$\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda).$$

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General depolarizing semigroup

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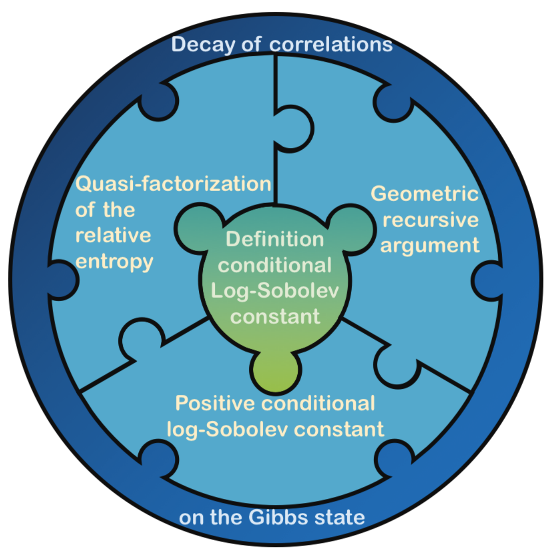
Strategy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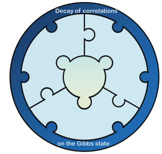
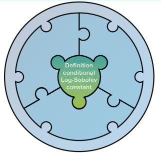
Heat-bath with tensor product fixed point



Assumption

$$\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x .$$

Heat-bath with tensor product fixed point



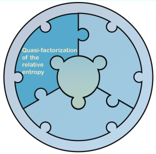
Conditional log-Sobolev constant

For $x \in \Lambda$, we define the **conditional log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ in x by

$$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_x(\rho_\Lambda || \sigma_\Lambda)},$$

where σ_Λ is the fixed point of the evolution, and $D_x(\rho_\Lambda||\sigma_\Lambda)$ is the conditional relative entropy.

Heat-bath with tensor product fixed point

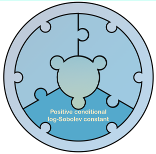


General quasi-factorization for σ a tensor product

Let $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$ such that $\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x$. The following inequality holds:

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda).$$

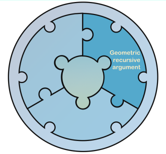
Heat-bath with tensor product fixed point



Lemma (Positivity of the conditional log-Sobolev constant)

$$\alpha_\Lambda(\mathcal{L}_x^*) \geq \frac{1}{2}.$$

Heat-bath with tensor product fixed point



$$
\begin{aligned}
D(\rho_\Lambda || \sigma_\Lambda) &\leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda) \\
&\leq \sum_{x \in \Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x^*)} \\
&\leq \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x \in \Lambda} - \operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \\
&= \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \left(- \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \right) \\
&\leq \left(- \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \right).
\end{aligned}
$$

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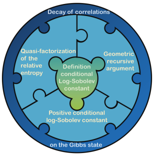

Positive log-Sobolev constant

$$\alpha(\mathcal{L}_\Lambda^*) \geq \frac{1}{2}.$$

Previous results:

- **Müller-Hermes et al. '15.** Lower bound 1/2 for the usual depolarizing semigroup, with fixed point $\mathbb{1}/d$.
- **Temme et al. '14.** For this semigroup, the log-Sobolev constant is positive, with a lower bound that is not universal.

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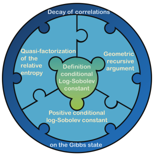

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Example 2, (Bardet-C.-Lucia-Pérez García-Rouzé, '19)

Heat-bath dynamics in 1D

Heat-bath dynamics in 1D

$\sigma_\Lambda = \frac{e^{-\beta H}}{\text{tr}(e^{-\beta H})}$ the Gibbs state of a k-local, commuting Hamiltonian H.

Consider the local and global Lindbladians

$$\mathcal{L}_x^* := \mathbb{E}_x^* - \mathbb{1}_\Lambda, \; \mathcal{L}_\Lambda^* = \sum_{x \in \Lambda} \mathcal{L}_x^*,$$

with

$$\mathbb{E}_x^*(\rho_\Lambda) = \sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2},$$

for every $\rho_\Lambda \in \mathcal{S}_\Lambda$,

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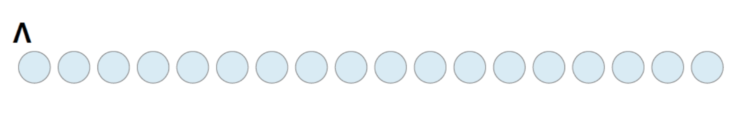
Log-Sobolev inequality for the heat-bath dynamics



The dynamics: For every $\rho_\Lambda \in \mathcal{S}_\Lambda$,

$$\mathcal{L}^*_\Lambda(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right).$$

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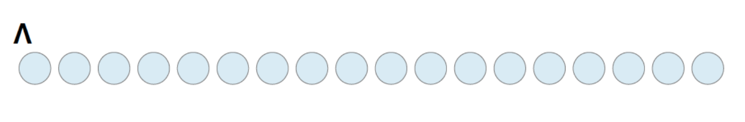

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$$\alpha(\mathcal{L}_\Lambda^*) \geq \Psi(A)\, \alpha_\Lambda(\mathcal{L}_A^*) \ ?$$

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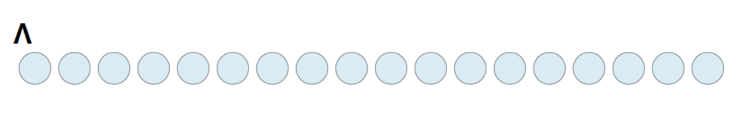

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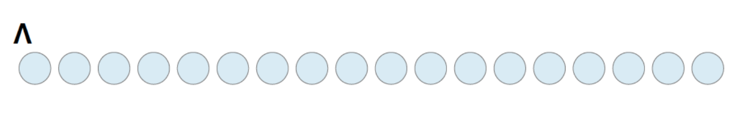

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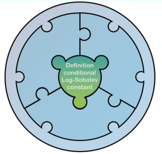
Heat-bath dynamics in 1D



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where σ_Λ is the fixed point of the evolution, and

$$D_A(\rho_\Lambda || \sigma_\Lambda) = D(\rho_\Lambda || \sigma_\Lambda) - D(\rho_{A^c} || \sigma_{A^c}).$$

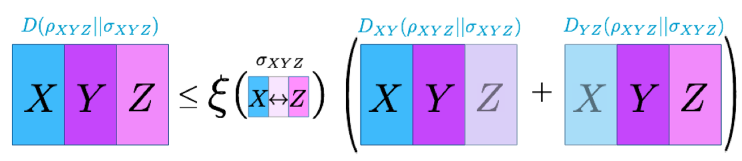
Quasi-factorization for the CRE

Let $\mathcal{H}_{XYZ}$ and $\rho_{XYZ}, \sigma_{XYZ} \in \mathcal{S}_{XYZ}$. The following holds

$$D(\rho_{XYZ}||\sigma_{XYZ}) \leq \xi(\sigma_{XZ}) \left[D_{XY}(\rho_{XYZ}||\sigma_{XYZ}) + D_{YZ}(\rho_{XYZ}||\sigma_{XYZ}) \right],$$

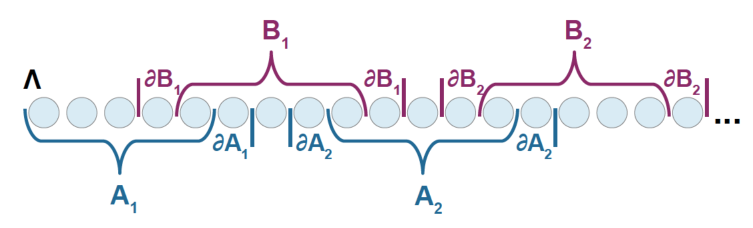
where

$$\xi(\sigma_{XZ}) = \frac{1}{1 - 2\left\| \sigma_X^{-1/2} \otimes \sigma_Z^{-1/2} \sigma_{XZ} \, \sigma_X^{-1/2} \otimes \sigma_Z^{-1/2} - \mathbb{1}_{XZ} \right\|_\infty}.$$



Quasi-factorization of the relative entropy

STEP 1



$$A = \bigcup_{i=1}^{n} A_i \text{ and } B = \bigcup_{j=1}^{n} B_j$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \frac{1}{1 - 2\| h(\sigma_{A^c B^c}) \|_\infty} \left[D_A(\rho_\Lambda || \sigma_\Lambda) + D_B(\rho_\Lambda || \sigma_\Lambda) \right],$$

$$h(\sigma_{A^c B^c}) := \sigma_{A^c}^{-1/2} \otimes \sigma_{B^c}^{-1/2} \sigma_{A^c B^c} \sigma_{A^c}^{-1/2} \otimes \sigma_{B^c}^{-1/2} - \mathbb{1}_{A^c B^c}.$$

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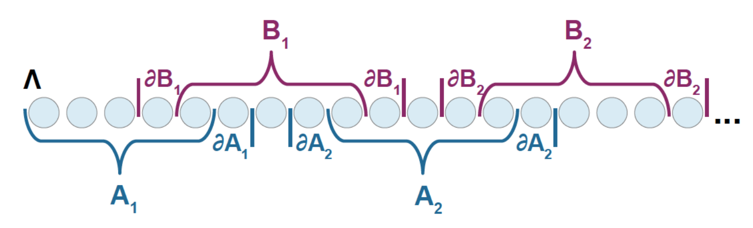

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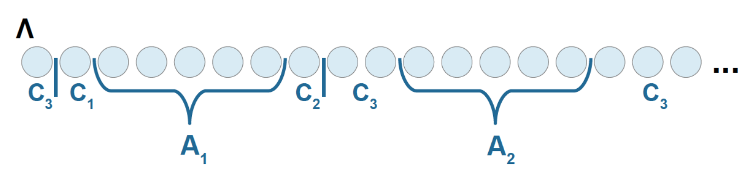
Sketch of the proof

STEP 2



$$D_A(\rho_\Lambda || \sigma_\Lambda) \leq \sum_{i=1}^{n} D_{A_i}(\rho_\Lambda || \sigma_\Lambda)$$

σ_Λ is a QMC between $A_1 \leftrightarrow \partial A_1 \leftrightarrow \Lambda \setminus (A_1 \cup \partial A_1)$

$$\sigma_\Lambda = \bigoplus_{i \in I} \sigma_{A_1 (\partial a_1)_i^L} \otimes \sigma_{(\partial a_1)_i^R \Lambda \setminus (A_1 \cup \partial A_1)}$$

Sketch of the proof

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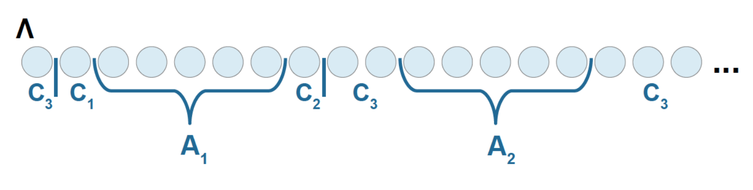

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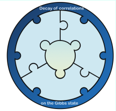
Heat-bath dynamics in 1D



Assumption 1

In a tripartite Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_C \otimes \mathcal{H}_B$, A and B not connected, we have

$$\left\| h(\sigma_{AB}) \right\|_\infty = \left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty \leq K < \frac{1}{2}.$$

In particular, Gibbs states at high-enough temperature satisfy this.

Assumption 2

For any $B \subset \Lambda$, $B = B_1 \cup B_2$, it holds:

$$D_B(\rho_\Lambda || \sigma_\Lambda) \leq f(\sigma_{B\partial}) \left(D_{B_1}(\rho_\Lambda || \sigma_\Lambda) + D_{B_2}(\rho_\Lambda || \sigma_\Lambda) \right).$$

In particular, tensor products satisfy this (with $f = 1$).

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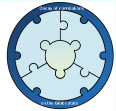

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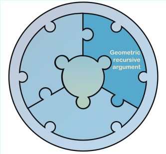


STEP 3

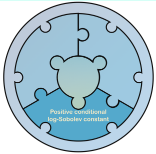
Using locality of the Lindbladian

$$\mathcal{L}_A^* + \mathcal{L}_B^* = \mathcal{L}_{A\cup B}^* + \mathcal{L}_{A\cap B}^*$$

and quasi-factorization:

$$\text{Assumption 1} \Rightarrow \alpha(\mathcal{L}_\Lambda^*) \geq \tilde{K} \min_{i\in\{1,\ldots n\}} \left\{ \alpha_\Lambda(\mathcal{L}_{A_i}^*), \alpha_\Lambda(\mathcal{L}_{B_i}^*) \right\}$$

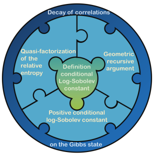
Heat-bath dynamics in 1D



STEP 4

$$\text{Assumption 2} \Rightarrow \alpha_\Lambda(\mathcal{L}^*_{A_i}) \geq g(\sigma_{A_i \partial}) > 0.$$

Heat-bath dynamics in 1D



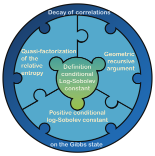
Theorem (Bardet-C.-Lucia-Pérez García-Rouzé, '19)

In 1D, if Assumptions 1 and 2 hold, for a k-local commuting Hamiltonian, the heat-bath dynamics has a positive log-Sobolev constant.

Previous results:

- **Kastoryano-Brandao, '15.** In 1D, for a k-local commuting Hamiltonian, the heat-bath dynamics is always gapped.

Heat-bath dynamics in 1D



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Example 3 (Bardet-C.-Rouzé, '20)

Davies dynamics

Davies dynamics

Generator

The generator of the Davies dynamics is of the following form:

$$\mathcal{L}_\Lambda^\beta(X) = i[H_\Lambda, X] + \sum_{k\in\Lambda} \mathcal{L}_k^\beta(X)\,,$$

where

$$\mathcal{L}_k^\beta(X) = \sum_{\omega,\alpha} \chi_{\alpha,k}^\beta(\omega) \left(S_{\alpha,k}^*(\omega) X S_{\alpha,k}(\omega) - \frac{1}{2}\left\{ S_{\alpha,k}^*(\omega) S_{\alpha,k}(\omega), X \right\} \right) .$$

Important property: Given $A \subseteq \Lambda$,

$$\mathcal{E}_A^\beta(X) := \mathcal{E}(X|\mathcal{N}_A) = \lim_{t\to\infty} \mathrm{e}^{t\mathcal{L}_A^\beta}(X)\,.$$

is a conditional expectation onto the subalgebra of fixed points of $\mathcal{L}_A^\beta$.

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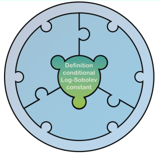

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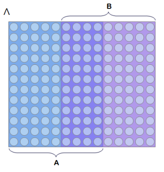
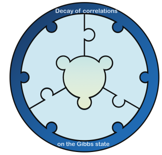
where σ_Λ is the fixed point of the global evolution (the Gibbs state of a local commuting Hamiltonian), and

$$D_A^\beta(\rho_\Lambda || \sigma_\Lambda) = D(\rho_\Lambda || (\mathcal{E}_A^\beta)^*(\rho_\Lambda)).$$

Davies dynamics





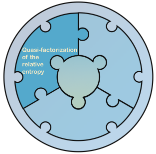
Exponential decay of correlations

If $\sigma \in \mathcal{S}(\mathcal{H})$ is a fixed point of the evolution and $f, g \in \mathcal{A}(\mathcal{H})$ such that $f \in \mathcal{A}_A$ and $g \in \mathcal{A}_B$, then

$$\left|\operatorname{tr}[\sigma f g] - \operatorname{tr}[\sigma f] \operatorname{tr}[\sigma g]\right| \leq c \|f\|_\infty \|g\|_\infty \, \mathrm{e}^{-d(A \setminus B, B \setminus A)} .$$

Spectral gap	**Log-Sobolev constant**
Change $\|\cdot\|_\infty \mapsto \|\cdot\|_{2,\sigma}$	Change $\|\cdot\|_\infty \mapsto \|\cdot\|_{1,\sigma}$
Conditional version	Conditional version
	Assume it for every fixed point

Davies dynamics

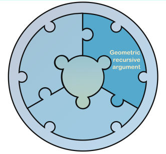


Quasi-factorization (Bardet-C.-Rouzé, '20)

Assume that there exists a constant $0 < c < \dfrac{1}{2(4+\sqrt{2})}$ such that there is exponential conditional $\mathbb{L}_1$-clustering of correlations with corresponding constant c. Then, the following inequality holds for every $\rho \in \mathcal{S}(\mathcal{H})$:

$$D^{\beta}_{AB}(\rho || \sigma) \leq \frac{1}{1-2(4+\sqrt{2})\, c} \left(D^{\beta}_{A}(\rho || \sigma) + D^{\beta}_{B}(\rho || \sigma) \right),$$

for every $\sigma = \mathcal{E}^{*}_{AB}(\sigma)$.



Geometric recursive argument

$$\alpha\left(\mathcal{L}_{\Lambda}^{\beta *}\right) \geq \Psi(L_0) \min_{R \in \mathcal{R}_{L_0}} \alpha_{\Lambda}\left(\mathcal{L}_{R}^{\beta^*}\right) ,$$

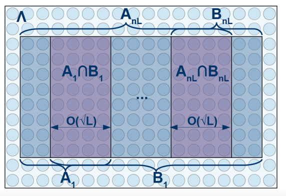
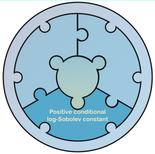


Figure: Splitting in A_n and B_n.



Conjecture

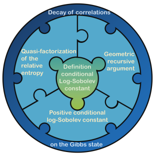
Given $\Lambda \subset\subset \mathbb{Z}^d$, $\mathcal{L}_\Lambda^* : \mathcal{S}_\Lambda \to \mathcal{S}_\Lambda$ the Lindbladian associated to the Davies dynamics and a finite lattice and $A \subset \Lambda$, we have

$$\alpha_\Lambda \left(\mathcal{L}_A^{\beta *} \right) \geq \psi(|A|) > 0,$$

where $\psi(|A|)$ might depend on Λ, but is independent of its size.

Uses **Junge et al. '19**.

Davies dynamics



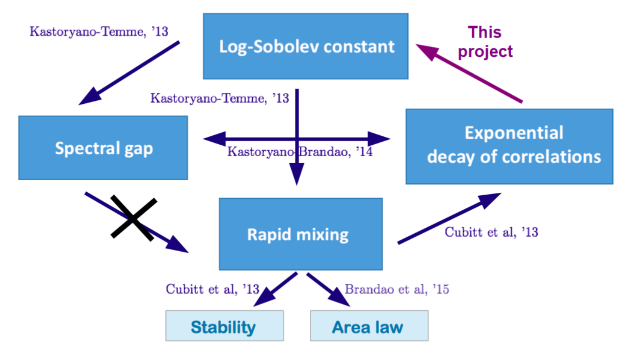
Theorem (Bardet-C.-Rouzé, '20)

Under exponential conditional $\mathbb{L}_1$-clustering of correlations, and assuming that the previous conjecture holds, for a k-local commuting Hamiltonian, the Davies dynamics has a positive log-Sobolev constant.

Previous results:

- **Kastoryano-Brandao, '15.** Under strong clustering, for a k-local commuting Hamiltonian, the Davies dynamics is gapped.

5. Conclusions



Open problems

Problem 1

Does the heat-bath result hold for larger dimension?

Problem 2

Is there a better definition for conditional relative entropy?

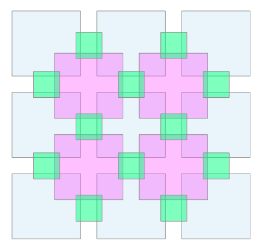
Problem 3

Can we do something similar for different dynamics?

Extension of log-Sobolev for heat-bath to larger dimensions

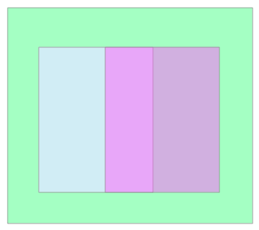
2 possible approaches:

- $D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC})\,(D_A + D_B + D_C)\,(\rho_{ABC}||\sigma_{ABC})$



Extension of log-Sobolev for heat-bath to larger dimensions

- $D_{AB}(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left(D_A(\rho_{ABC}||\sigma_{ABC}) + D_B(\rho_{ABC}||\sigma_{ABC}) \right)$



Other approaches

- In **(Bardet-C.-Rouzé, '20)**, we deal with **approximate tensorization**, namely:

$$D^{\beta}_{AB}(\rho||\sigma) \leq c \left(D^{\beta}_{A}(\rho||\sigma) + D^{\beta}_{B}(\rho||\sigma) \right) + d \, ,$$

- In an ongoing project **(C.-Rouzé-Stilck França, '20)**, we consider instead **global approximate tensorization**:

$$D^{\beta}_{A}(\rho||\sigma) \leq c \sum_{x \in A} D^{\beta}_{x}(\rho||\sigma) \, ,$$

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thank you
danke
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Баярлалаа
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