Rapid thermalization of spin chain commuting Hamiltonians

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Cambyse Rouzé (T. U. München)

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Open quantum systems

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Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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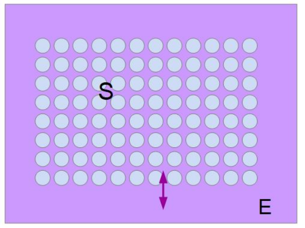
⇒ Open quantum many-body systems.



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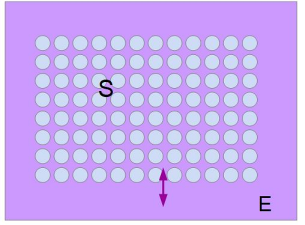

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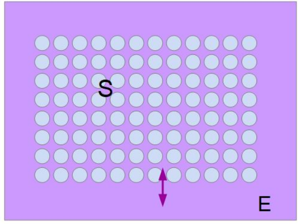

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$\Lambda \subset \mathbb{Z}^d$ a finite lattice.

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A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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The infinitesimal generator $\mathcal{L}_\Lambda^*$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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For $\rho_\Lambda \in \mathcal{S}_\Lambda$, $\quad \mathcal{L}_\Lambda^*(\rho_\Lambda) = -i[H_\Lambda, \rho_\Lambda] + \sum_{k\in\Lambda} L_k^*(\rho_\Lambda) \quad .$

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Rapid mixing

$\Lambda \subset \mathbb{Z}^d$ a finite lattice.

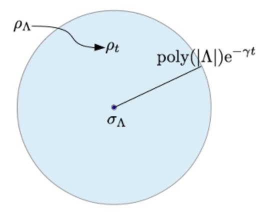
Notation: $\rho_t := \mathcal{T}_t^*(\rho)$ and σ_Λ the unique fixed-point.

$$\rho_\Lambda \xrightarrow{t} \rho_t := \mathcal{T}_t^*(\rho_\Lambda) = e^{t\mathcal{L}_\Lambda^*}(\rho_\Lambda) \xrightarrow{t\to\infty} \sigma_\Lambda$$

Rapid mixing

We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$



Modified log-Sobolev inequality (MLSI)

Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

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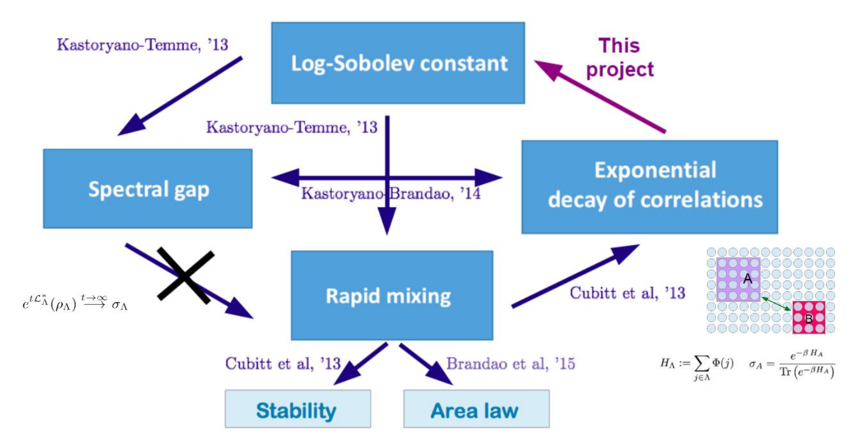


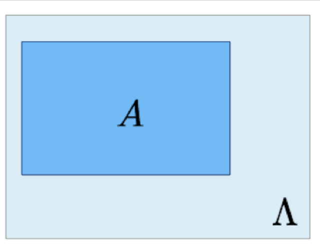
Exp. decay of correlations:

$$\sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]| \leq K\, \mathrm{e}^{-\gamma d(A,B)} \quad .$$

Objective

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}^*_\Lambda) \geq \Psi(|\Lambda|) > 0.$$



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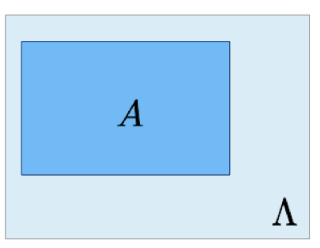

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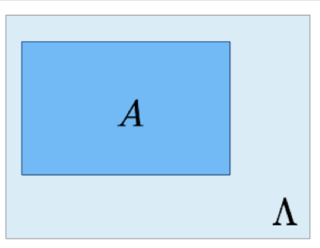

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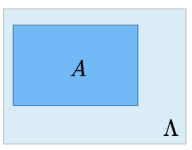

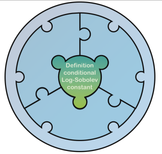

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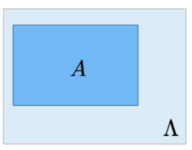

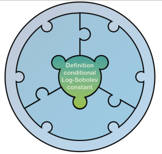

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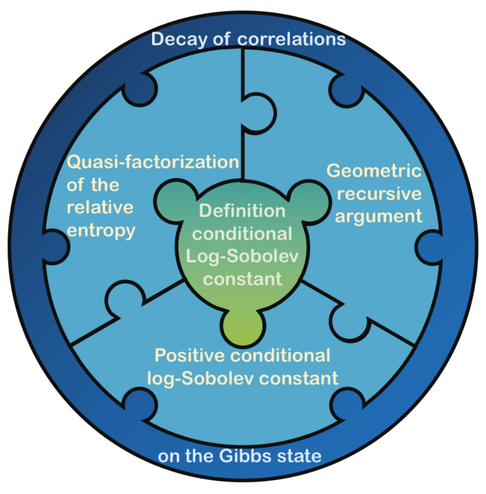
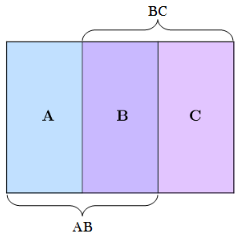
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

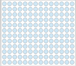
$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Example: Tensor product fixed point

(C.-Lucia-Pérez García '18) $\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$ **heat-bath**

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

$$\leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda)$$

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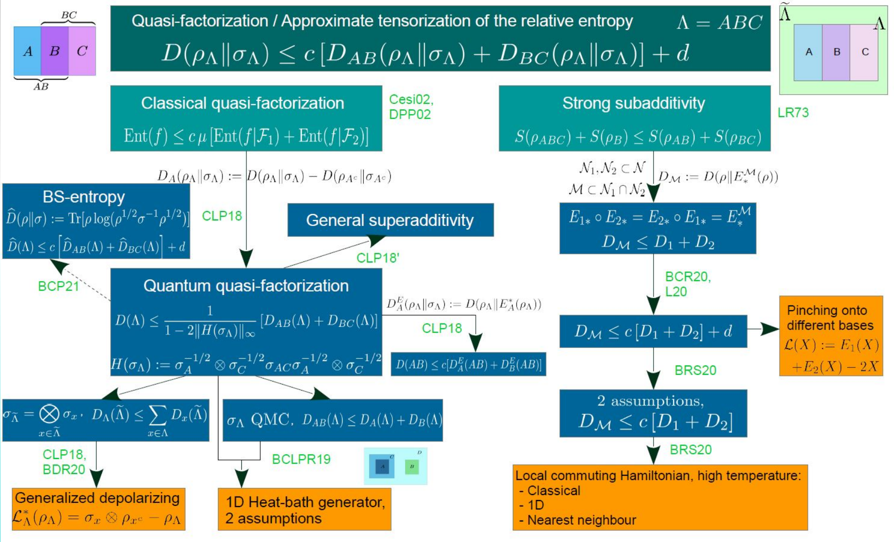
$$\leq \sum_{x \in \Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 \alpha_\Lambda(\mathcal{L}_x^*)}$$

$$\leq \frac{1}{2 \inf_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x \in \Lambda} - \operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]$$

$$= \frac{1}{2 \inf_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \left(- \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \right)$$

$$\leq \left(- \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \right) . \qquad \Rightarrow \alpha(\mathcal{L}_\Lambda^*) \geq 1/2.$$

QUASI-FACTORIZATION OF THE RELATIVE ENTROPY



Main result

MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '21)

Let $\mathcal{L}_\Lambda^*$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^*$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^*) = \Omega(\ln(|\Lambda|)^{-1})$.

Rapid mixing:

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

Main result

MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '21)

Let $\mathcal{L}_\Lambda^*$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^*$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^*) = \Omega(\ln(|\Lambda|)^{-1})$.

Rapid mixing:

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For $\alpha(\mathcal{L}_\Lambda^*)$ a **MLSI constant**:

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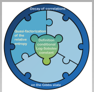
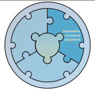
Proof: Quasi-factorization + Geometric recursion





$\sigma \equiv \sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\mathrm{tr}(e^{-\beta H_\Lambda})}$ Gibbs state of local, comm., t-i Ham.

Cond. relative entropies: $D_X(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{X^c} \| \sigma_{X^c})$,

$$D(\rho_\Lambda \| E_X(\rho_\Lambda)) \quad \text{with} \quad E_X(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2} \sigma_{X^c}^{-1/2} \mathrm{tr}_X[\,\cdot\,] \sigma_{X^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n .$$

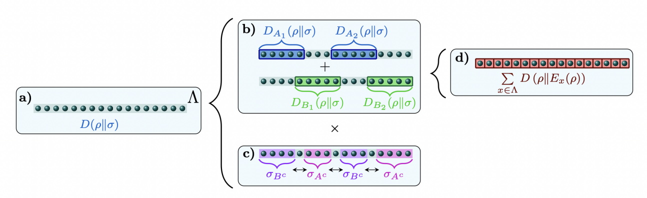
Quasi-factorization + Decay of correlations

Let $(\cup_i A_i) \cup (\cup_i B_i) = \Lambda \subset \mathbb{Z}$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$. The following holds

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \mathcal{K} \sum_i \left[D_{A_i}(\rho_\Lambda || \sigma_\Lambda) + D_{B_i}(\rho_\Lambda || \sigma_\Lambda) \right]$$

$$\leq \widetilde{\mathcal{K}} \sum_{x \in \Lambda} \left[D(\rho_\Lambda || E_x(\rho_\Lambda)) \right],$$

where $\mathcal{K}$ is constant as long as # segments $= \mathcal{O}(|\Lambda| / \ln |\Lambda|)$ and $\widetilde{\mathcal{K}} = O(\log |\Lambda|)$.



Proof: Positive conditional MLSI

MLSI and conditional MLSI

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Therefore, we have

$$\alpha(\mathcal{L}_\Lambda^*) \geq \widetilde{\mathcal{K}}^{-1} \min_{x \in \Lambda} \{\alpha_x(\mathcal{L}_\Lambda^*)\}.$$

for $\widetilde{\mathcal{K}}^{-1} = \Omega(\ln|\Lambda|^{-1})$.

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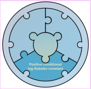

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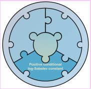

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