Rapid thermalization of spin chain commuting Hamiltonians

Modified logarithmic Sobolev inequalities for quantum many-body systems

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Joint work with **I. Bardet, L. Gao, A. Lucia, D. Pérez-García, C. Rouzé**

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Motivation: Open quantum many-body systems

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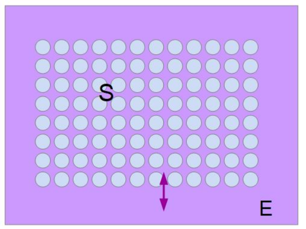

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x\in\Lambda} \mathcal{H}_x$.
- Density matrices: $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \, : \, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

- Dynamics of S is dissipative!
- The continuous-time evolution of a state on S is given by a q. Markov semigroup (**Markovian approximation**).

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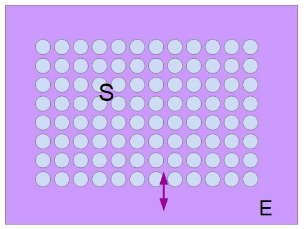

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Quantum Markov semigroup / Dissipative quantum evolution

Quantum Markov semigroup

A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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The infinitesimal generator $\mathcal{L}_\Lambda$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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For $\rho_\Lambda \in \mathcal{S}_\Lambda$, $\quad \mathcal{L}_\Lambda(\rho_\Lambda) = -i[H_\Lambda, \rho_\Lambda] + \sum_{k\in\Lambda} \widetilde{\mathcal{L}}_k(\rho_\Lambda) \quad$ **GKLS equation**.

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Mixing $\Leftrightarrow$ Convergence

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We also assume that the quantum Markov process studied is **reversible**, i.e., it satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}_\Lambda^*(g) \rangle_\sigma = \langle \mathcal{L}_\Lambda^*(f), g \rangle_\sigma ,$$

for every $f, g \in \mathcal{B}_\Lambda$ and Hermitian, where

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Mixing time

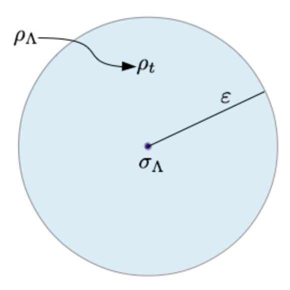
- Under the previous conditions, there is always convergence to σ_Λ.
- How fast does convergence happen?

Note $\mathcal{T}_\infty(\rho) := \sigma_\Lambda$ for every ρ.

Mixing time

We define the **mixing time** of $\{\mathcal{T}_t\}$ by

$$t_{\text{mix}}(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t(\rho) - \mathcal{T}_\infty(\rho) \right\|_1 \leq \varepsilon \right\}.$$



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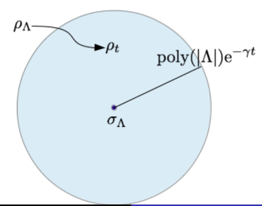
Rapid mixing

We say that $\mathcal{L}_\Lambda$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

$t_{\text{mix}}(\varepsilon) \sim \text{poly} \log(|\Lambda|)$.



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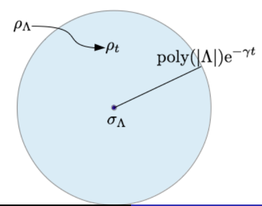

Applications to quantum information/quantum computing

What are the implications of rapid mixing?

Rapid mixing

$$\sup_{\rho \in \mathcal{S}(\mathcal{H}_\Lambda)} \|T_t(\rho) - \sigma\|_1 \leq \text{poly}(|\Lambda|)\mathrm{e}^{-\gamma t}$$

Mixing time: $\tau(\varepsilon) = \mathcal{O}(\text{polylog}(|\Lambda|))$

"Negative" point of view:

- Quantum properties that hold in the ground state but not in the Gibbs state are **suppressed too fast** for them to be of any reasonable use.

"Positive" point of view:

- Thermal states with short mixing time can be **constructed efficiently** with a quantum device that simulates the effect of the thermal bath.
- This has important implications as a self-studying open problem as well as in optimization problems via simulated annealing type algorithms.

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If **rapid mixing**, no **error correction:**

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Easy $t_{\text{mix}} \sim \log(n)$ — $t_{\text{mix}} \sim \text{poly}(n)$ — $t_{\text{mix}} \sim \exp(n)$ **Hard**

Efficient prediction

Speed-up for SDP solvers

Error correction
Topological order

Self-correction
Quantum memories

Main applications or consequences:

- Robust and efficient **preparation of topologically ordered phases** of matter via dissipation.
- Design of more efficient **quantum error-correcting codes** optimized for correlated Markovian noise models.
- **Stability** against local perturbations (Cubitt, Lucia, Michalakis, Pérez-García '15)
- **Area law** for mutual information (Brandao, Cubitt, Lucia, Michalakis, Pérez-García '15)
- Gaussian **concentration inequalities** (Lipschitz observables) (C., Rouzé, S. Franca '20)
- Finite blocklength refinement of **quantum Stein lemma** (C., Rouzé, Stilck Franca '20)
- **Quantum annealers:** Output an energy closed to that of the fixed point after short time (C., Rouzé, Stilck Franca '20)
- **Preparation Gibbs states:** Existence of local quantum circuits with logarithmic depth to prepare the Gibbs state (C., Rouzé, Stilck Franca '20)
- Establish the absence of **dissipative phase transitions** (Bardet, C., Gao, Lucia, Pérez-García, Rouzé '21)
- Examples of interacting **SPT phases** with decoherence time growing logarithmically with the system size for thermal noise (Bardet, C., Gao, Lucia, Pérez-García, Rouzé '21)

And many more...

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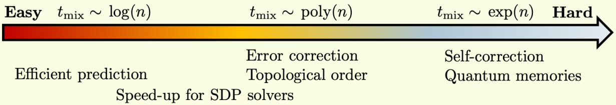

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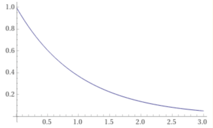

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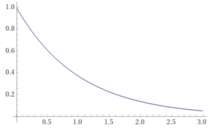

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If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) > 0$:

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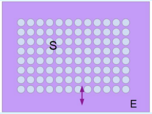

Notation: $\Lambda \subset\subset \mathbb{Z}^d$ lattice

$\{T_t\}_{t \geq 0}$ Quantum Markov semigroup

$\mathcal{L}$ Inf. generator (Lindbladian)

Mixing time of the semigroup $\{T_t\}_{t \geq 0}$

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Quantum spin systems

Kastoryano-Temme, '13

Modified Logarithmic Sobolev Inequality (MLSI)

$$D(T_t(\rho)\|\sigma) \leq D(\rho\|\sigma)\, \mathrm{e}^{-2\alpha(\mathcal{L})t}$$

Relative entropy: $D(\rho\|\sigma) := \mathrm{tr}[\rho(\log\rho - \log\sigma)]$

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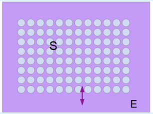

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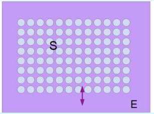

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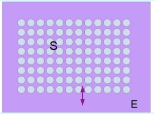

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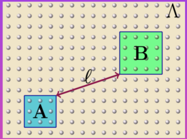
Decay of correlations



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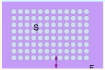

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Decay of correlations on Gibbs state

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial B}$,
- **Gibbs state:** $\sigma_\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} \,/\, \mathrm{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.

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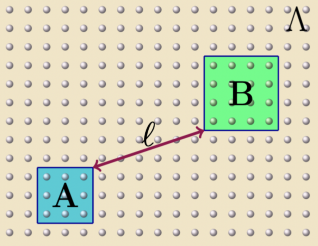

$\ell := \mathrm{dist}(A, B)$

Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta H_{A\cup B}} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \; ?$$

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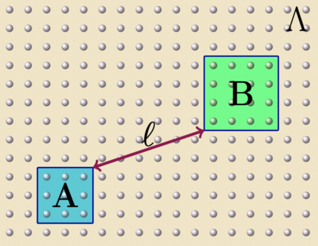

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Decay of correlations on Gibbs state

3 different forms of **decay of correlations**.

Operator correlation

$$\mathrm{Cov}_\sigma(A:B) := \sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]|$$

Mutual information

$$I_\sigma(A:B) := D(\sigma_{AB}||\sigma_A \otimes \sigma_B)$$

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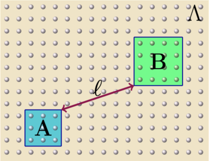

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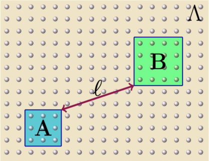

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Quantum spin systems

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KT'13

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Cubitt et al.'13

KT'13

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Kastoryano-Brandao'14

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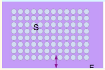

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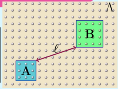

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C et al.'13

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KB'14

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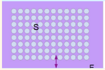

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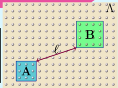

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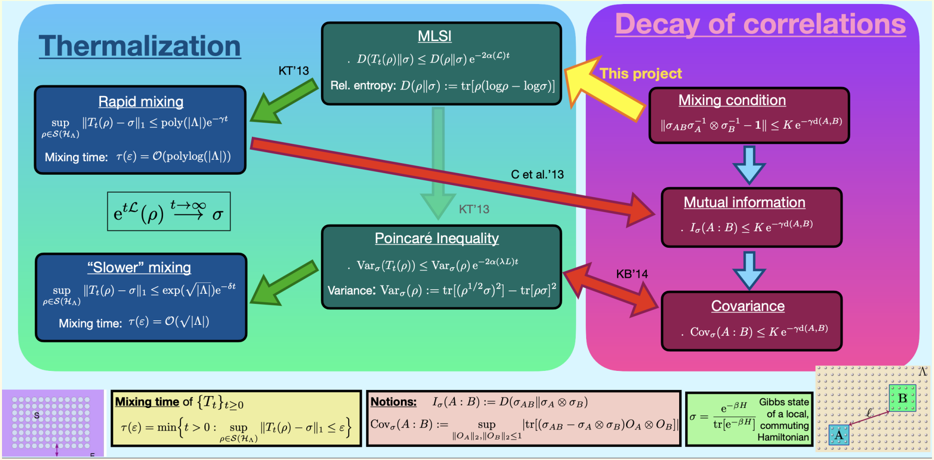

Mathematical Challenge in Quantum Physics

Given:

- H_Λ local (commuting) Hamiltonian $\quad\mapsto\quad \sigma_\Lambda := \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}[e^{-\beta H_\Lambda}]}$ Gibbs state .
- $\mathcal{L}_\Lambda$ local Lindbladian with unique stationary state σ_Λ ($\mathcal{L}_\Lambda(\sigma_\Lambda) = 0$).

Questions:

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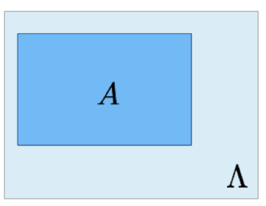
MLSI constant

$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) \geq \Psi(|\Lambda|) > 0 \qquad \left(\text{or} = 0 \text{ very "slowly", like } \frac{1}{\text{poly} \log(|\Lambda|)}\right)$$



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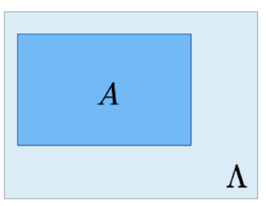

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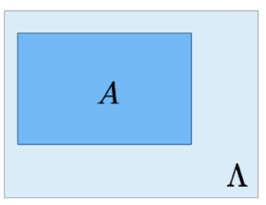

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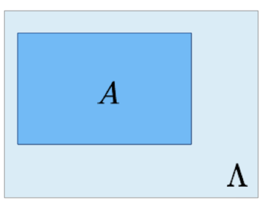

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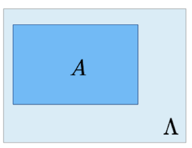
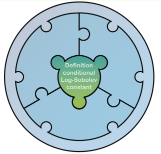
Conditional MLSI constant





MLSI constant

The **MLSI constant** of $\mathcal{L}_\Lambda = \sum_{k \in \Lambda} \mathcal{L}_k$ is defined by

$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

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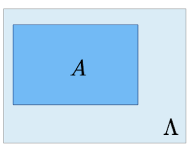

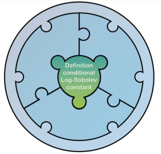

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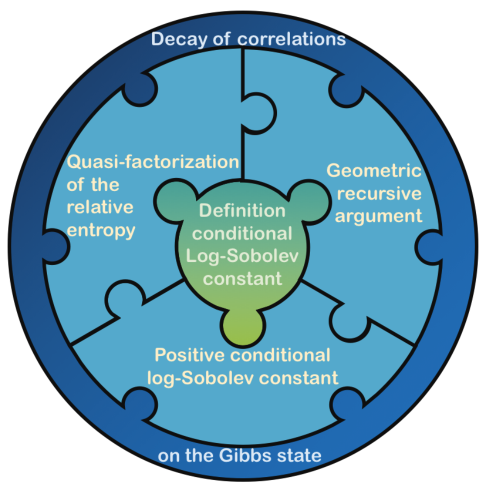
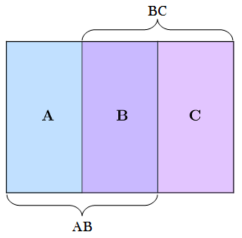
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

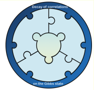
How does the strategy work?

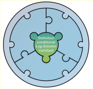
We want to prove:

$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda \| \sigma_\Lambda)}$

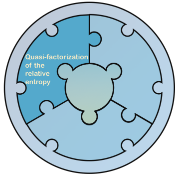
$$\boxed{\alpha(\mathcal{L}_\Lambda) \geq \Psi(|A|)\, \alpha_\Lambda(\mathcal{L}_A) > 0}$$

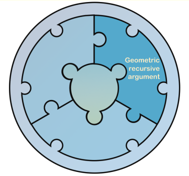
$\alpha_\Lambda(\mathcal{L}_A) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_A(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_A(\rho_\Lambda \| \sigma_\Lambda)}$

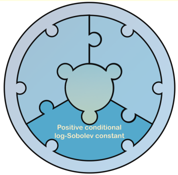
After choosing 

and 

, we prove the following:







$D(\rho_\Lambda \| \sigma_\Lambda) \to D_A(\rho_\Lambda \| \sigma_\Lambda)$ $\qquad \Psi(|A|) > 0$ $\qquad \alpha_\Lambda(\mathcal{L}_A) > 0$

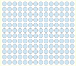
Example: Tensor product fixed point

(C.-Lucia-Pérez García '18)

(Beigi-Datta-Rouzé '18)

$$\mathcal{L}_\Lambda(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda) \qquad \textbf{heat-bath}$$

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

$$\leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda)$$

$$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 D_x(\rho_\Lambda || \sigma_\Lambda)}$$

$$\leq \sum_{x \in \Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 \alpha_\Lambda(\mathcal{L}_x)}$$

$$\leq \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x)} \sum_{x \in \Lambda} - \operatorname{tr}[\mathcal{L}_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]$$

$$= \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x)} \left(- \operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right)$$

$$\leq \left(- \operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right) .$$

Dynamics

Let $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}\left[e^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

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The **Davies generator** is given by:

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where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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Previous results

Let us recall: For $\alpha(\mathcal{L}_\Lambda)$ a MLSI constant,

$$\|\rho_t - \sigma_\Lambda\|_1 \leq \sqrt{2\log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda)\, t}.$$

Using the spectral gap $\lambda(\mathcal{L}_\Lambda)$:

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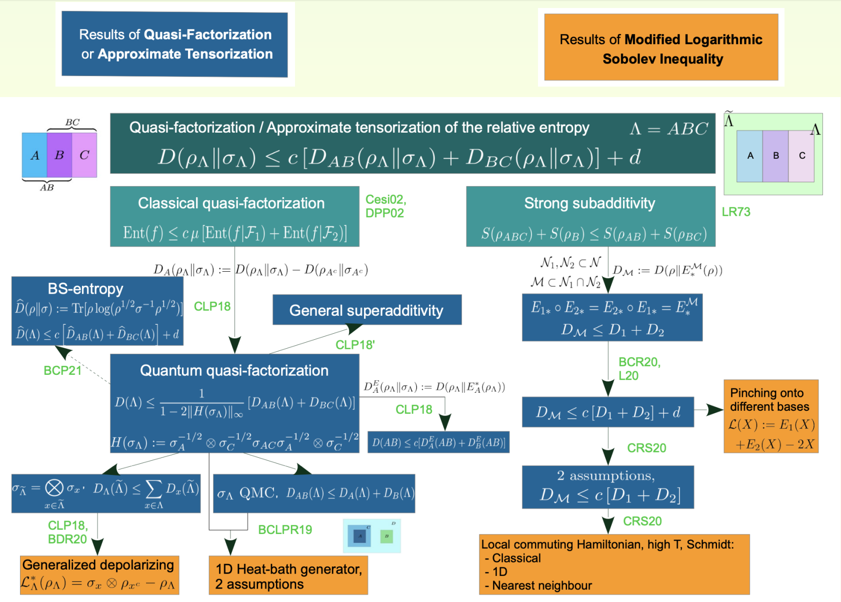

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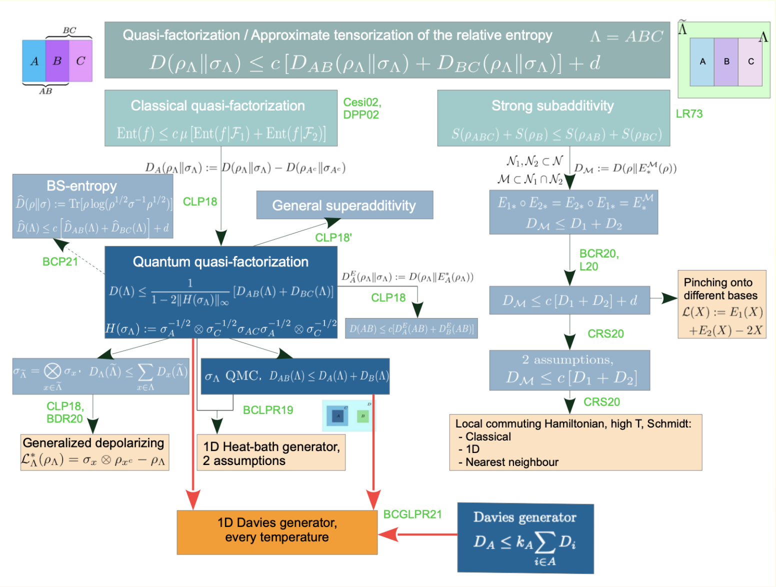

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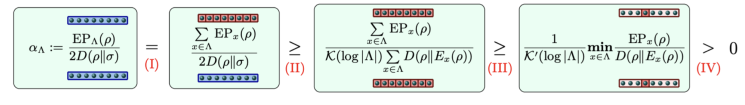

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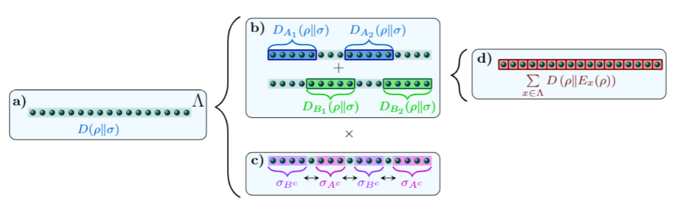

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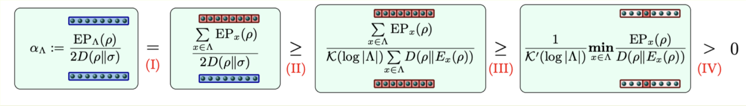

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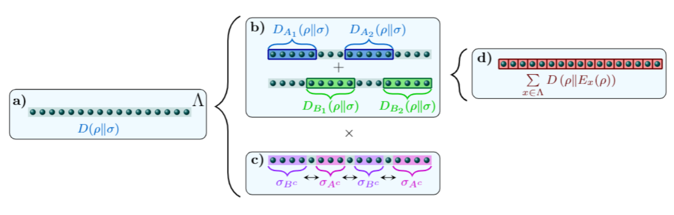

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Important: Our result does not apply in the presence of a strong symmetry.

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- **Dissipative phase transitions:** Absence of dissipative phase transitions in 1D for Davies evolutions over translation-invariant spin chains.
- **Symmetry Protected Topological phases:** Example of a non-trivial interacting SPT phase with decoherence time of $O(\log |\Lambda|)$.

Corollary for SPT phases

For every $\beta > 0$, 1D SPT phases thermalize in time logarithmic in $|\Lambda|$, even when the thermal bath is chosen to be weakly symmetric.

Example: 1D Cluster state. Unique ground state of a Hamiltonian with 3-local interactions given by $Z \otimes X \otimes Z$ (and p.b.c.).

Important: Our result does not apply in the presence of a strong symmetry.

Conclusions

In this talk:

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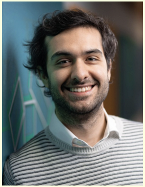
Thank you for your attention!

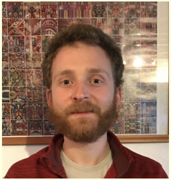
Do you have any questions?

David Pérez-García
U. Complutense
Madrid

Angelo Lucia
U. Complutense
Madrid



Cambyse Rouzé
T. U. Munich



Ivan Bardet
Inria Paris

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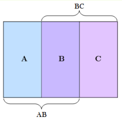
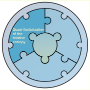
UNED Madrid

Andreas Bluhm
U. Grenoble

Li Gao
U. Houston

Proof: Conditional relative entropies + Quasi-factorization





Conditional relative entropies: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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Heat-bath cond. expectation: $E_A(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2} \sigma_{A^c}^{-1/2} \operatorname{tr}_A[\,\cdot\,] \, \sigma_{A^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n$.

Quasi-factorization (C.-Lucia-Pérez García '18)

Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

$$D(\rho_{ABC} \| \sigma_{ABC}) \leq \xi(\sigma_{AC}) \left[D_{AB}(\rho_{ABC} \| \sigma_{ABC}) + D_{BC}(\rho_{ABC} \| \sigma_{ABC}) \right],$$

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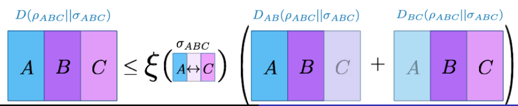
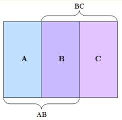
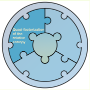


Proof: Conditional relative entropies + Quasi-factorization





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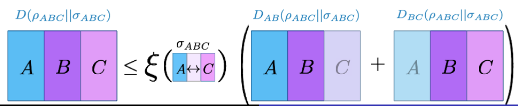

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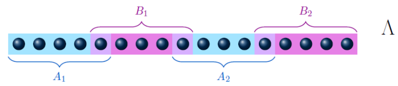

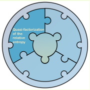

$\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}(e^{-\beta H_\Lambda})}$ is the Gibbs state of a k-local, commuting Hamiltonian H_Λ.

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Quasi-factorization for quantum Markov chains (Bardet-C.-Lucia-Pérez García-Rouzé'19)

Since σ_Λ is a QMC between $A_i \leftrightarrow \partial(A_i) \leftrightarrow (A_i \cup \partial A_i)^c$, then:

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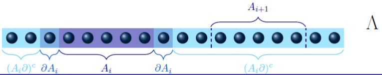

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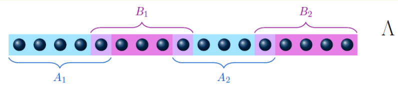

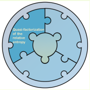

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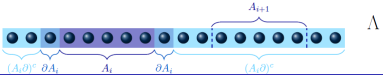
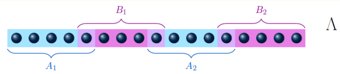


Proof: Decay of correlations



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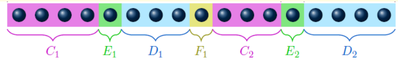

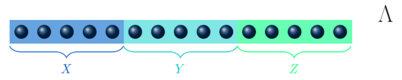

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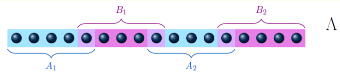

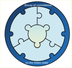

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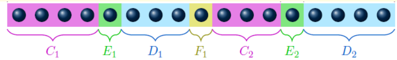

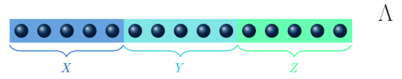

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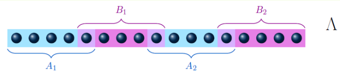

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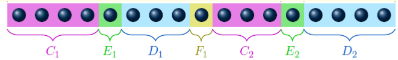

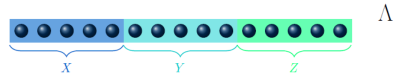

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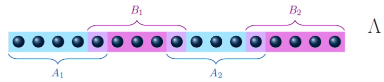
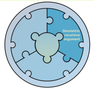
Proof: Geometric recursive argument





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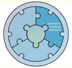
Comparison conditional rel. ent. (Bardet-C.-Rouzé, '20)

$$D_A(\rho_\Lambda \| \sigma_\Lambda) \leq D_A^E(\rho_\Lambda \| \sigma_\Lambda)$$

Therefore, by this and  +  , we have:

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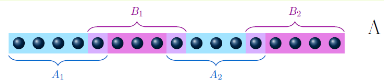

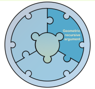

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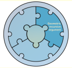
Therefore, by this and  +  , we have:

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Proof: Positive CMLSI

Reduction of cond. relative entropies (Gao-Rouzé, '21)

$$D(\rho_\Lambda \| E_{A_i}(\rho_\Lambda)) \leq 4 k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda} \, ,$$

where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

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As a consequence of the non-closure of the spectral gap proved for 1D commuting Gibbs samplers (Kastoryano-Brando '16), $k_{A_i} = \mathcal{O}(\ln |\Lambda|)$ for $A_i = \mathcal{O}(\ln |\Lambda|)$.

Proof: Positive CMLSI

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Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda} \, ,$$

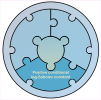
where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

As a consequence of the non-closure of the spectral gap proved for 1D commuting Gibbs samplers (Kastoryano-Brando '16), $k_{A_i} = \mathcal{O}(\ln |\Lambda|)$ for $A_i = \mathcal{O}(\ln |\Lambda|)$.

CMLSI (Gao-Rouzé, '21)

The CMLSI of the local generators is positive:

$$\alpha_c(\mathcal{L}_j^D) := \inf_{k \in \mathbb{N}} \alpha(\mathcal{L}_j^D \otimes \mathrm{Id}_k) > 0 \, .$$



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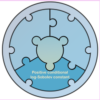

Last step

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Davies cond. expectation: $E_A^D(\cdot) := \lim_{t \to \infty} \mathrm{e}^{t\mathcal{L}_A^D}(\cdot) .$

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Conclusion

For $\mathcal{L}_\Lambda^D$, there is a positive MLSI constant $\alpha(\mathcal{L}_\Lambda^D) = \Omega(\ln|\Lambda|^{-1})$.
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