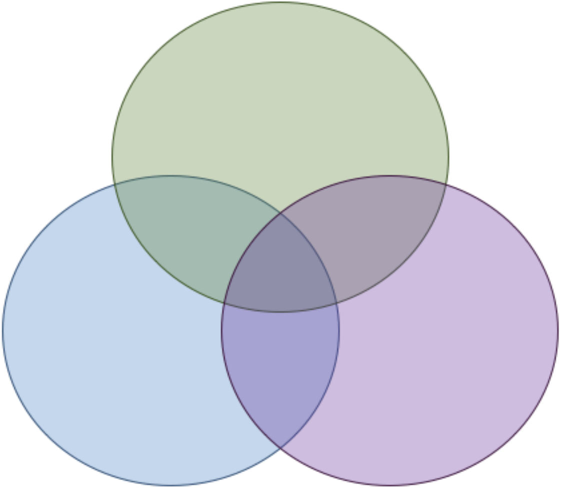
Logarithmic Sobolev Inequalities for Quantum Many-Body Systems

Ángela Capel Cuevas (ICMAT)

Joint work with Ivan Bardet (INRIA, Paris), Angelo Lucia (Caltech), Cambyse Rouzé (T. U. München) and David Pérez-García (U. Complutense de Madrid).

Granada, 4 December 2019

Field of study



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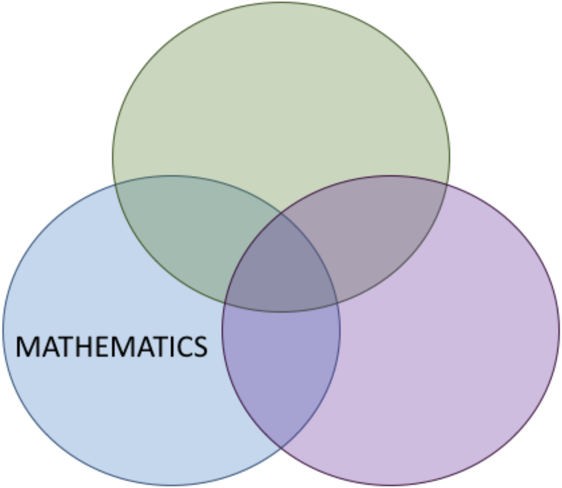

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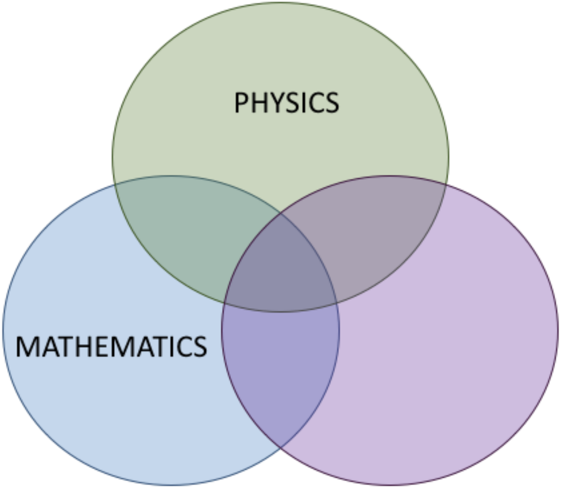

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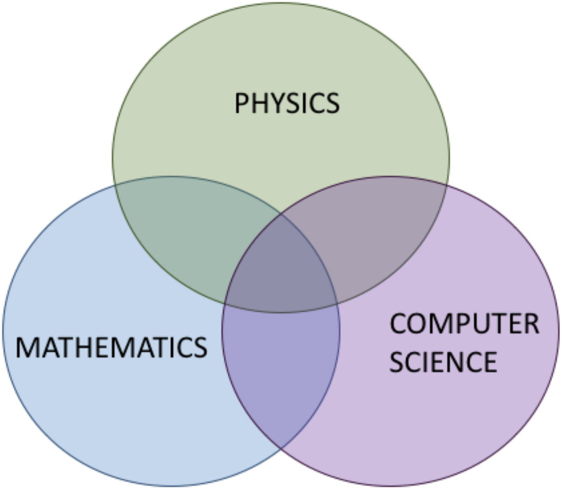

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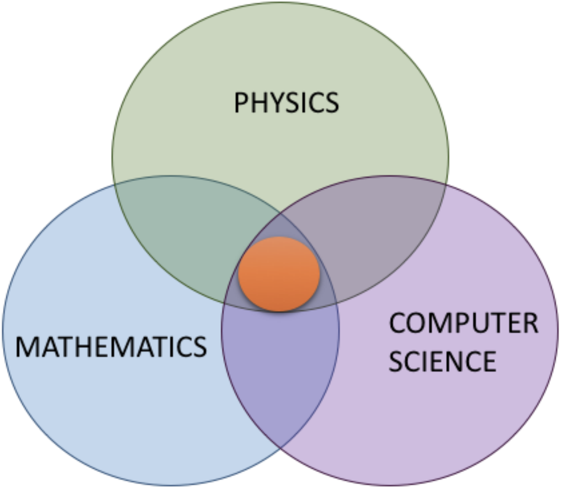

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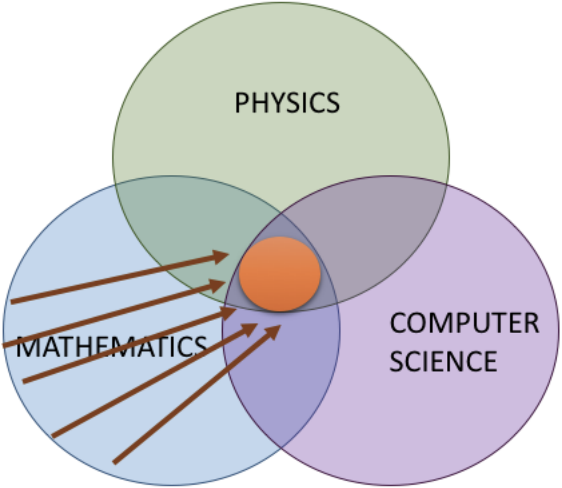

Quantum

Q. information theory $\longleftrightarrow$ Q. many-body physics

Communication channels $\longleftrightarrow$ Physical interactions

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Dissipative evolutions of quantum many-body systems

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Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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Provide sufficient static conditions on a Gibbs state which imply the existence of a positive log-Sobolev constant.

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Contents

1.1 Quantum dissipative systems

Open quantum systems

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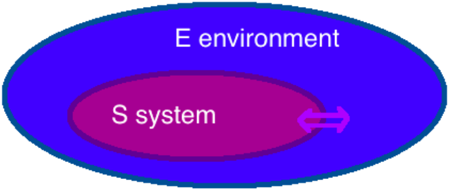
$\Rightarrow$ Open quantum many-body systems.



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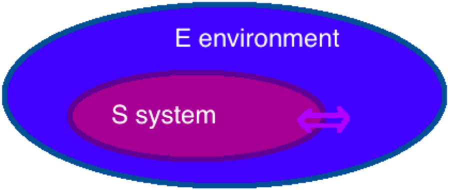

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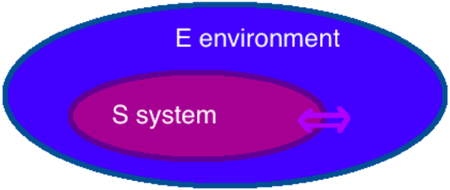

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Postulates of quantum mechanics

Postulate 1

Given an isolated physical system, there is a complex Hilbert space $\mathcal{H}$ associated to it, which is known as the **state space** of the system.

Moreover, the physical system is completely described by its **state vector**, which is a unitary vector in the state space.

Postulate 2

Given an isolated physical system, its evolution is described by a **unitary transformation** in the Hilbert space.

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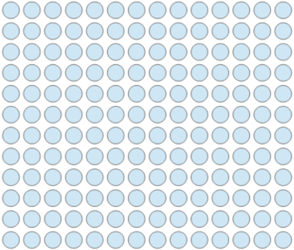

Figure: A quantum spin lattice system.

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- To every site $x \in \Lambda$ we associate $\mathcal{H}_x$ $(= \mathbb{C}^D)$.
- The global Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x\in\Lambda} \mathcal{H}_x$.
- The set of bounded linear endomorphisms on $\mathcal{H}_\Lambda$ is denoted by $\mathcal{B}_\Lambda := \mathcal{B}(\mathcal{H}_\Lambda)$.
- The set of density matrices is denoted by $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \ : \ \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

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Isolated system.

Physical evolution: $\rho \mapsto U\rho U^* \rightsquigarrow$ Reversible

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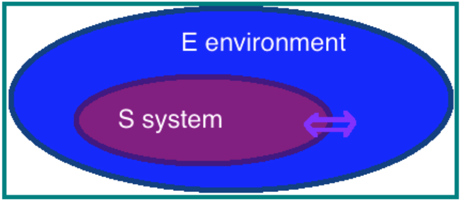
Open systems ⇒ Environment and system interact.



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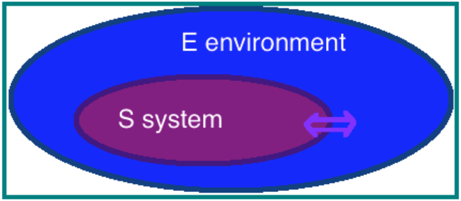

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State for the environment: $|\psi\rangle\langle\psi|_E$

$$\rho \mapsto \rho \otimes |\psi\rangle\langle\psi|_E \mapsto U\left(\rho \otimes |\psi\rangle\langle\psi|_E\right)U^* \mapsto \mathrm{tr}_E[U\left(\rho \otimes |\psi\rangle\langle\psi|_E\right)U^*] = \tilde{\rho}$$

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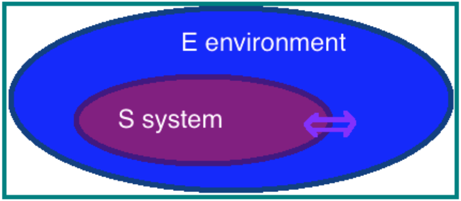

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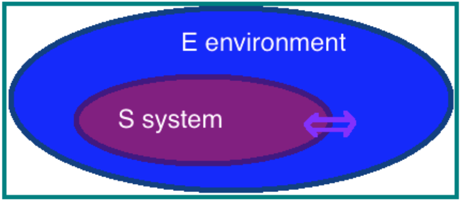

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Continuous-time description: For every $t \geq 0$, the corresponding time slice is a realizable evolution $\mathcal{T}_t$ (quantum channel).

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A **dissipative quantum system** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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QMS generator

The infinitesimal generator $\mathcal{L}_\Lambda^*$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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Primitive QMS

We assume that $\{\mathcal{T}_t^*\}_{t\geq 0}$ has a unique full-rank invariant state, which we denote by σ.

Reversibility

We also assume that the quantum Markov process studied is **reversible**, i.e., satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}(g)\rangle_\sigma = \langle \mathcal{L}(f), g\rangle_\sigma$$

for every $f, g \in \mathcal{A}$, in the Heisenberg picture.

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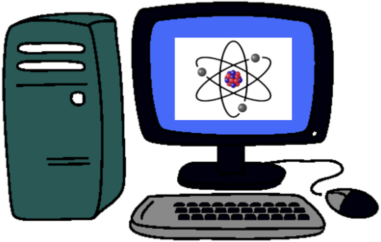
Quantum dissipative evolutions useful?

Main objective:



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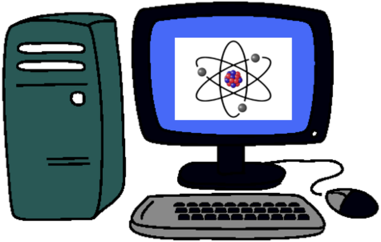

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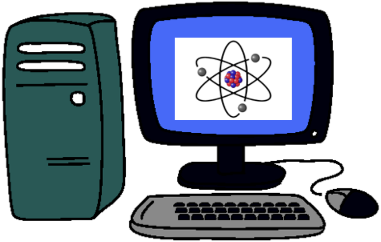

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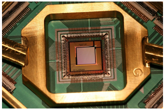

Recent change of perspective $\Rightarrow$ Resource to exploit

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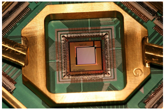

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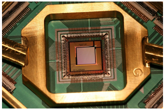

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Mixing time

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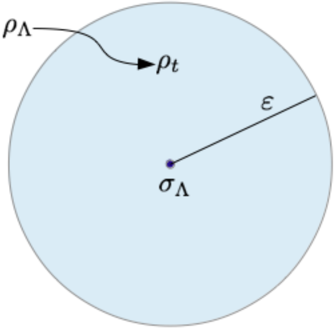
We define the **mixing time** of $\{\mathcal{T}_t^*\}$ by

$$\tau(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t^*(\rho) - \mathcal{T}_\infty^*(\rho) \right\|_1 \leq \varepsilon \right\}.$$



Rapid mixing

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We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

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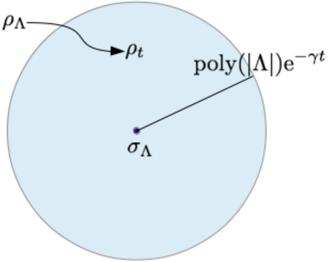

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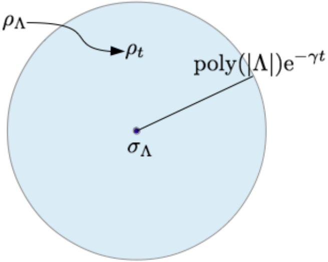

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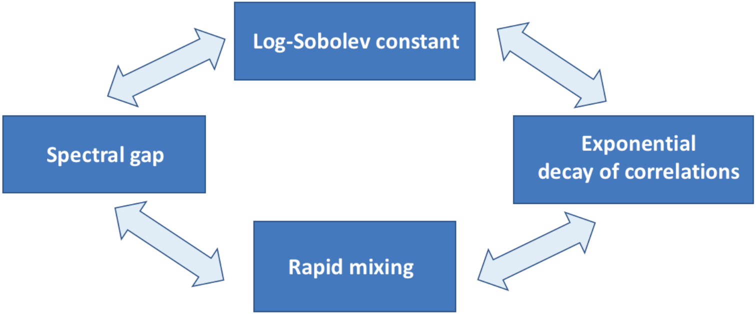
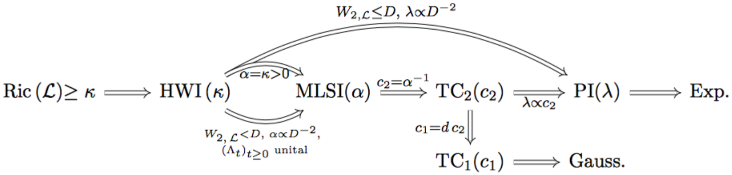
1.2 Logarithmic Sobolev inequalities

Classical spin systems



Quantum spin systems



Log-Sobolev inequality (MLSI)

Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

Liouville's equation:

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$$D(\rho_t || \sigma_\Lambda) = \operatorname{tr}[\rho_t(\log \rho_t - \log \sigma_\Lambda)].$$

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Differentiating:

$$\partial_t D(\rho_t||\sigma_\Lambda) = \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)]. \tag{1}$$

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$$D(\rho_t || \sigma_\Lambda) = \operatorname{tr}[\rho_t(\log \rho_t - \log \sigma_\Lambda)].$$

Differentiating:

$$\partial_t D(\rho_t || \sigma_\Lambda) = \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)]. \tag{1}$$

Lower bound for the derivative of $D(\rho_t || \sigma_\Lambda)$ in terms of itself:

$$2\alpha D(\rho_t || \sigma_\Lambda) \leq -\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)]. \tag{2}$$

Log-Sobolev inequality (MLSI)

Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

Liouville's equation:

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The **log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ is defined as:

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If $\alpha(\mathcal{L}_\Lambda^*) > 0$:

$$D(\rho_t || \sigma_\Lambda) \leq D(\rho_\Lambda || \sigma_\Lambda) e^{-2\,\alpha(\mathcal{L}_\Lambda^*)\, t},$$

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$$\left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \sqrt{2D(\rho_\Lambda || \sigma_\Lambda)}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t} \leq \sqrt{2\log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t}.$$

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First main objective of this talk

Develop a strategy to find positive log Sobolev constants from static properties on the fixed point.

Second main objective of this talk

Apply that strategy to certain dissipative dynamics.

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2 Strategy to find log-Sobolev constants

Classical spin systems

(Cesi, Dai Pra-Paganoni-Posta, '02)

(1) Quasi-factorization of the entropy (in terms of a conditional entropy).

+

(2) Recursive geometric argument.
Lower bound for the global log-Sobolev constant in terms of the log-Sobolev constant of a size-fixed region.

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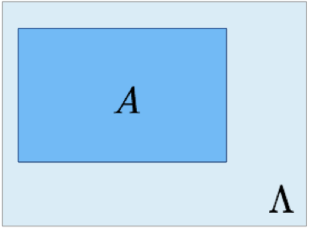
$\Downarrow$

Positive log-Sobolev constant.

Objective

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|\Lambda|) > 0.$$



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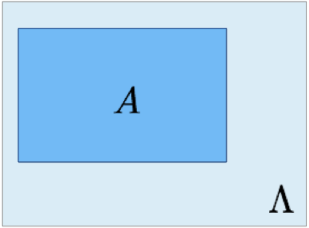

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Log-Sobolev constant

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Let $\mathcal{L}_\Lambda^* : \mathcal{S}_\Lambda \to \mathcal{S}_\Lambda$ be a primitive reversible Lindbladian with stationary state σ_Λ, $A \subseteq \Lambda$. We define the **conditional log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ on A by

$$\alpha_\Lambda(\mathcal{L}_A^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_A^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_A(\rho_\Lambda || \sigma_\Lambda)}$$

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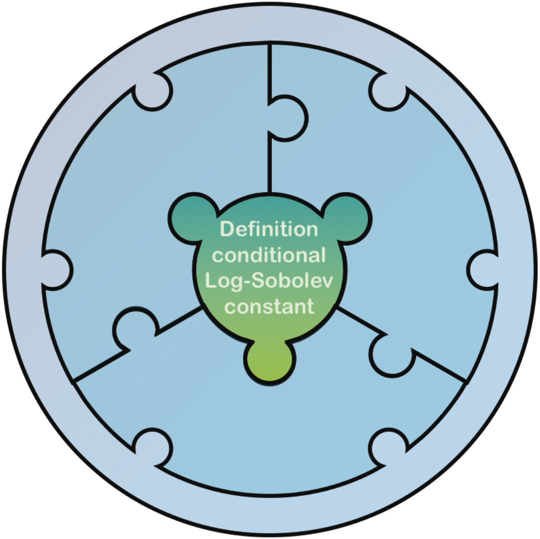

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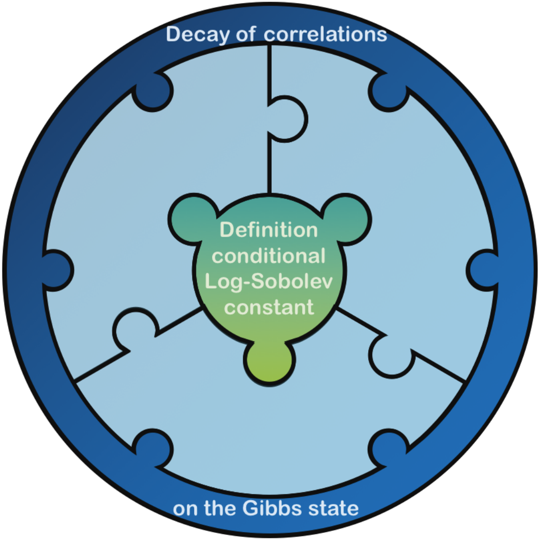
Decay of correlations
Definition
conditional
Log-Sobolev
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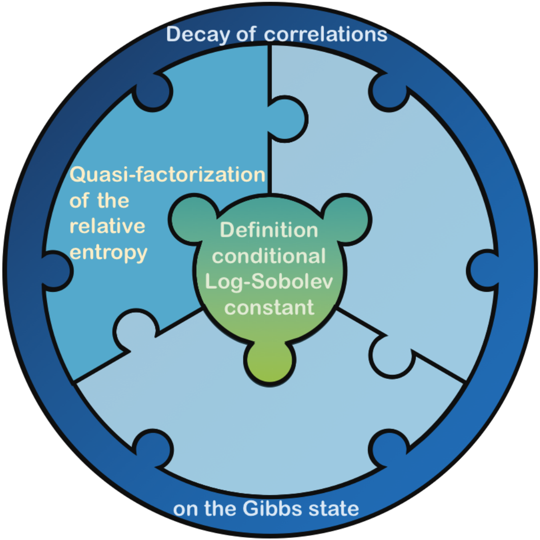
Decay of correlations
Quasi-factorization of the relative entropy
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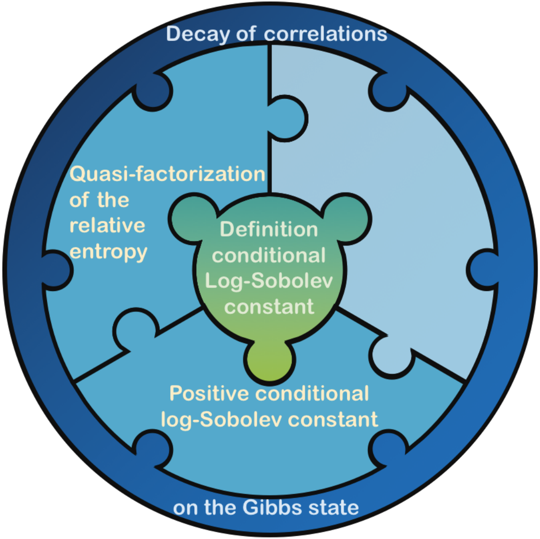
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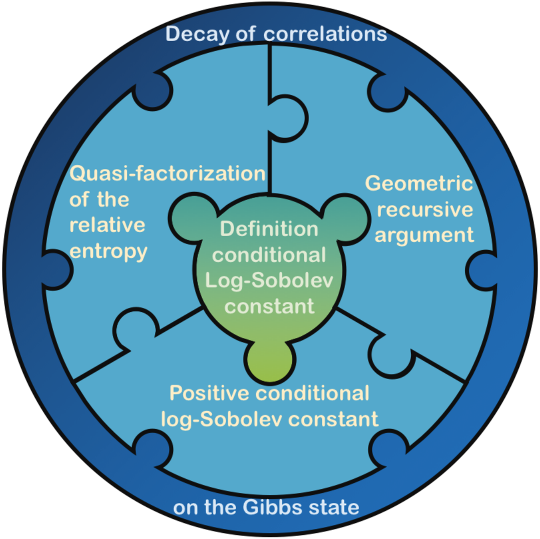
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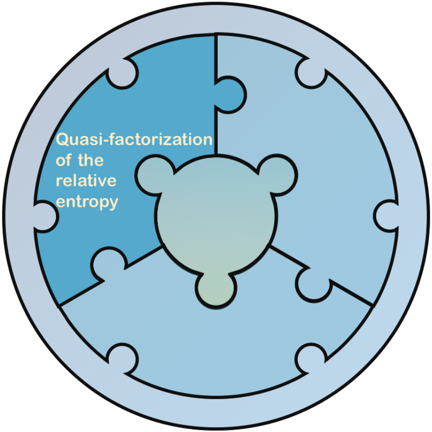
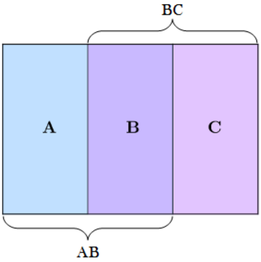
3 Quasi-factorization of the relative entropy



Statement of the problem



Problem

Let $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ and $\rho_{ABC}, \sigma_{ABC} \in S_{ABC}$. Can we prove something like

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right] \ ?$$

Quantum relative entropy

$$D(\rho||\sigma) = \operatorname{tr}\left[\rho(\log\rho - \log\sigma)\right]$$

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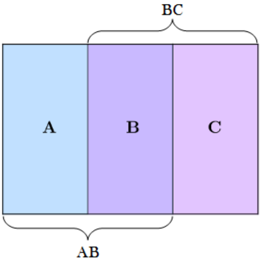

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Classical case, Dai Pra et al. '02

$$\mathrm{Ent}_\mu(f) \leq \frac{1}{1 - 4\|h-1\|_\infty} \, \mu \left[\mathrm{Ent}_\mu(f \mid \mathcal{F}_1) + \mathrm{Ent}_\mu(f \mid \mathcal{F}_2) \right],$$

where $h = \dfrac{d\mu}{d\bar{\mu}}$.

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Classical entropy and conditional entropy

Entropy:

$$\mathrm{Ent}_\mu(f) = \mu(f \log f) - \mu(f) \log \mu(f).$$

Conditional entropy:

$$\mathrm{Ent}_\mu(f \mid \mathcal{G}) = \mu(f \log f \mid \mathcal{G}) - \mu(f \mid \mathcal{G}) \log \mu(f \mid \mathcal{G}).$$

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Relative entropy

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Let $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$. The **quantum relative entropy** of ρ_Λ and σ_Λ is defined by:

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Properties of the relative entropy

Let $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ and $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$. The following properties hold:

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Characterization of the RE, Wilming et al. '17, Matsumoto '10

If $f : \mathcal{S}_{AB} \times \mathcal{S}_{AB} \to \mathbb{R}_0^+$ satisfies $1 - 4$, then f is the relative entropy.

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Conditional relative entropy

Conditional relative entropy, (Q-Fact)

Let $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$. We define a **conditional relative entropy** in A as a function

$$D_A(\cdot||\cdot) : \mathcal{S}_{AB} \times \mathcal{S}_{AB} \to \mathbb{R}_0^+$$

verifying the following properties for every $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$:

1. **Continuity:** The map $\rho_{AB} \mapsto D_A(\rho_{AB}||\sigma_{AB})$ is continuous.
2. **Non-negativity:** $D_A(\rho_{AB}||\sigma_{AB}) \geq 0$ and
   (2.1) $D_A(\rho_{AB}||\sigma_{AB})=0$ if, and only if, $\rho_{AB} = \sigma_{AB}^{1/2}\sigma_B^{-1/2}\rho_B\sigma_B^{-1/2}\sigma_{AB}^{1/2}$.
3. **Semi-superadditivity:** $D_A(\rho_{AB}||\sigma_A \otimes \sigma_B) \geq D(\rho_A||\sigma_A)$ and
   (3.1) **Semi-additivity:** if $\rho_{AB} = \rho_A \otimes \rho_B$, $D_A(\rho_A \otimes \rho_B||\sigma_A \otimes \sigma_B) = D(\rho_A||\sigma_A)$.
4. **Semi-motonicity:** For every quantum channel $\mathcal{T}$,

$$D_A(\mathcal{T}(\rho_{AB})||\mathcal{T}(\sigma_{AB})) + D_B((\operatorname{tr}_A \circ \mathcal{T})(\rho_{AB})||(\operatorname{tr}_A \circ \mathcal{T})(\sigma_{AB})) \leq D_A(\rho_{AB}||\sigma_{AB}) + D_B(\operatorname{tr}_A(\rho_{AB})|| \operatorname{tr}_A(\sigma_{AB})).$$

Remark

Consider for every $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$

$$D^+_{A,B}(\rho_{AB}||\sigma_{AB}) = D_A(\rho_{AB}||\sigma_{AB}) + D_B(\rho_{AB}||\sigma_{AB}).$$

Then, $D^+_{A,B}$ verifies the following properties:

1. **Continuity:** $\rho_{AB} \mapsto D^+_{A,B}(\rho_{AB}||\sigma_{AB})$ is continuous.
2. **Additivity:** $D^+_{A,B}(\rho_A \otimes \rho_B||\sigma_A \otimes \sigma_B) = D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)$.
3. **Superadditivity:** $D^+_{A,B}(\rho_{AB}||\sigma_A \otimes \sigma_B) \geq D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)$.

However, it does not satisfy the property of monotonicity.

Axiomatic characterization of the CRE, **(Q-Fact)**

The only possible conditional relative entropy is given by:

$$D_A(\rho_{AB}||\sigma_{AB}) = D(\rho_{AB}||\sigma_{AB}) - D(\rho_B||\sigma_B)$$

for every $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$.

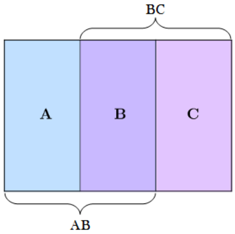

Figure: Choice of indices in $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$.

Result of **quasi-factorization** of the relative entropy, for every $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$:

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Quasi-factorization for the CRE, (Q-Fact)

Let $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. Then, the following inequality holds

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \frac{1}{1 - 2\|H(\sigma_{AC})\|_\infty} \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where

$$H(\sigma_{AC}) = \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \, \sigma_{AC} \, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC}.$$

Note that $H(\sigma_{AC}) = 0$ if σ_{AC} is a tensor product between A and C.

$$
\begin{aligned}
(1 - 2\|H(\sigma_{AC})\|_\infty) D(\rho_{ABC}||\sigma_{ABC}) \leq \\
D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) = \\
= 2D(\rho_{ABC}||\sigma_{ABC}) - D(\rho_C||\sigma_C) - D(\rho_A||\sigma_A).
\end{aligned}
$$

$$\Leftrightarrow$$

$$(1 + 2\|H(\sigma_{AC})\|_\infty) D(\rho_{ABC}||\sigma_{ABC}) \geq D(\rho_A||\sigma_A) + D(\rho_C||\sigma_C).$$

$$
\begin{aligned}
(1-2\|H(\sigma_{AC})\|_{\infty})D(\rho_{ABC}||\sigma_{ABC}) \leq \\
D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) = \\
= 2D(\rho_{ABC}||\sigma_{ABC}) - D(\rho_{C}||\sigma_{C}) - D(\rho_{A}||\sigma_{A}).
\end{aligned}
$$

$$\Leftrightarrow$$

$$(1+2\|H(\sigma_{AC})\|_{\infty})D(\rho_{ABC}||\sigma_{ABC}) \geq D(\rho_{A}||\sigma_{A}) + D(\rho_{C}||\sigma_{C}).$$

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\begin{aligned}
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This result is equivalent to **(Super)**:

$$\boxed{(1 + 2\|H(\sigma_{AB})\|_\infty) D(\rho_{AB}||\sigma_{AB}) \geq D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)}.$$

Recall:

- **Superadditivity.** $D(\rho_{AB}||\sigma_A \otimes \sigma_B) \geq D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)$.

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Due to:

- **Monotonicity.** $D(\rho_{AB}||\sigma_{AB}) \geq D(T(\rho_{AB})||T(\sigma_{AB}))$ for every quantum channel T.

we have

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Quasi-factorization for the CRE (Q-Fact)

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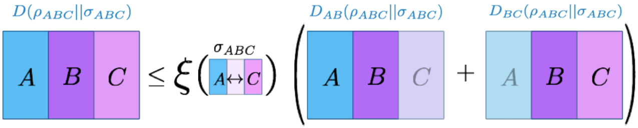
$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{AC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where

$$\xi(\sigma_{AC}) = \frac{1}{1 - 2\left\| \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \sigma_{AC}\, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC} \right\|_\infty}.$$



Relation with the classical case

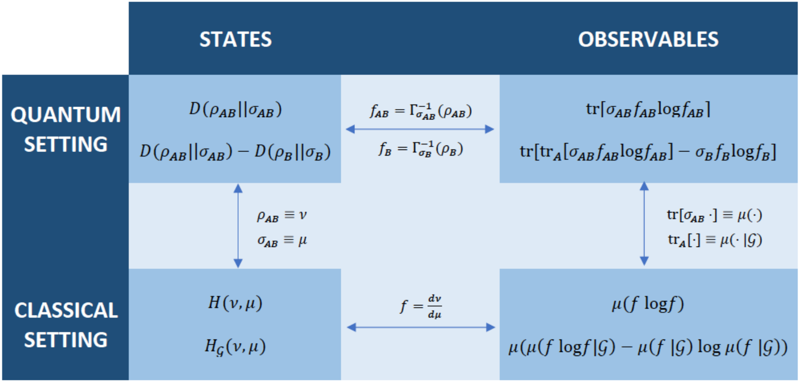
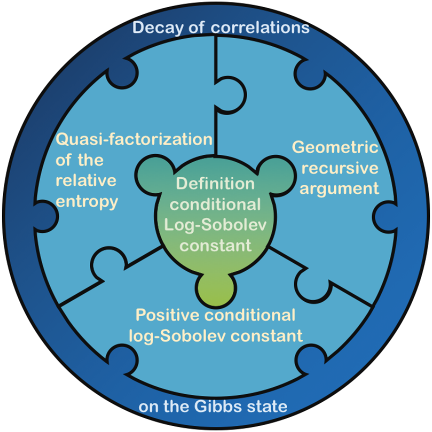


Figure: Identification between classical and quantum quantities when the states considered are classical.

4 Log-Sobolev constants



Quantum spin lattices

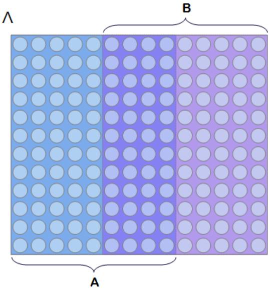


Figure: A quantum spin lattice system Λ and $A, B \subseteq \Lambda$ such that $A \cup B = \Lambda$.

Problem

For a certain $\mathcal{L}_\Lambda^*$, can we prove $\alpha(\mathcal{L}_\Lambda^*) > 0$ using the result of quasi-factorization of the relative entropy?

Example 1 (Q-Fact)

Heat-bath dynamics with tensor product fixed point

Heat-bath with tensor product fixed point

Theorem (Q-Fact)

The **heat-bath dynamics**, with tensor product fixed point, has a positive log-Sobolev constant.

Consider the local and global Lindbladians

$$\mathcal{L}_x^* := \mathbb{E}_x^* - \mathbb{1}_\Lambda, \; \mathcal{L}_\Lambda^* = \sum_{x \in \Lambda} \mathcal{L}_x^*$$

Since

$$\mathbb{E}_x^*(\rho_\Lambda) = \sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} = \sigma_x \otimes \rho_{x^c}$$

for every $\rho_\Lambda \in \mathcal{S}_\Lambda$, we have

$$\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda).$$

Heat-bath with tensor product fixed point

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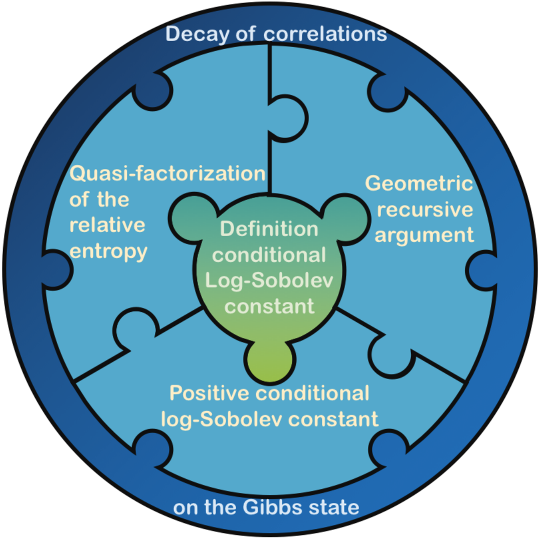
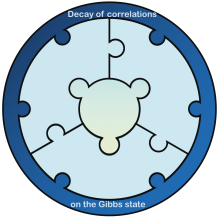
Strategy


Decay of correlations
Quasi-factorization of the relative entropy
Geometric recursive argument
Definition conditional Log-Sobolev constant
Positive conditional log-Sobolev constant
on the Gibbs state

Heat-bath with tensor product fixed point

Assumption

$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x .$$



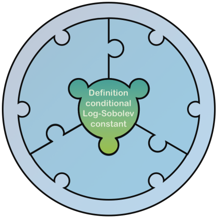
Heat-bath with tensor product fixed point

Conditional log-Sobolev constant

For $x \in \Lambda$, we define the **conditional log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ in x by

$$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_x(\rho_\Lambda || \sigma_\Lambda)},$$

where σ_Λ is the fixed point of the evolution, and $D_x(\rho_\Lambda || \sigma_\Lambda)$ is the conditional relative entropy.

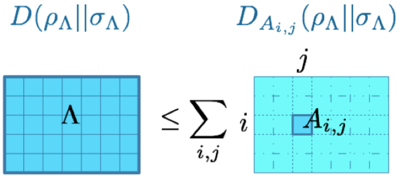


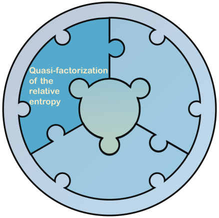
Heat-bath with tensor product fixed point

General quasi-factorization for σ a tensor product

Let $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$ such that $\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x$. The following inequality holds:

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda).$$

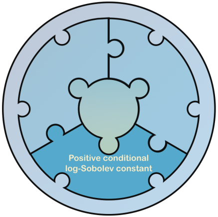




Heat-bath with tensor product fixed point

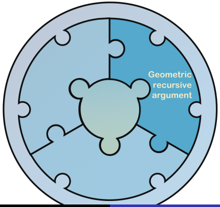
Lemma (Positivity of the conditional log-Sobolev constant)

$$\alpha_\Lambda(\mathcal{L}_x^*) \geq \frac{1}{2}.$$



Heat-bath with tensor product fixed point

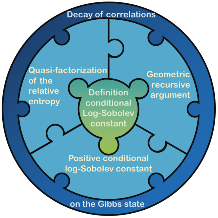
$$
\begin{aligned}
D(\rho_\Lambda || \sigma_\Lambda) &\leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda) \\
&\leq \sum_{x \in \Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x^*)} \\
&\leq \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x \in \Lambda} -\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \\
&= \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right) \\
&\leq \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right).
\end{aligned}
$$



Heat-bath with tensor product fixed point

Positive log-Sobolev constant

$$\alpha(\mathcal{L}_\Lambda^*) \geq \frac{1}{2}.$$



Example 2, (Heat-bath)

Heat-bath dynamics in 1D

Heat-bath dynamics in 1D

σ_Λ is the Gibbs state of a k-local, commuting Hamiltonian.

$\Phi : \Lambda \to \mathcal{A}_\Lambda$ be a k-local potential: For $j \in \Lambda$, $\Phi(j)$ self-adjoint and supported on a ball of radius k around site j.

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Assume: $\|\Phi(j)\| \leq K$ for some constant $K < \infty$. The potential Φ is said to be **commuting** if for any $i, j \in \Lambda$, $[\Phi(i), \Phi(j)] = 0$.

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$$H_A := \sum_{j \in A} \Phi(j) \,. \tag{3}$$

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$$\sigma_A^\beta := \frac{e^{-\beta H_A}}{\operatorname{tr}(e^{-\beta H_A})} \, . \tag{4}$$

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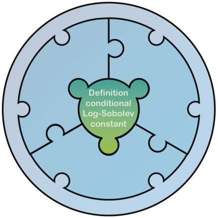
where σ_Λ is the fixed point of the evolution, and

$$D_A(\rho_\Lambda||\sigma_\Lambda) = D(\rho_\Lambda||\sigma_\Lambda) - D(\rho_{A^c}||\sigma_{A^c}).$$



Quasi-factorization for the CRE (Q-Fact)

Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{AC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where

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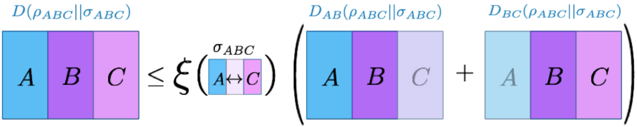
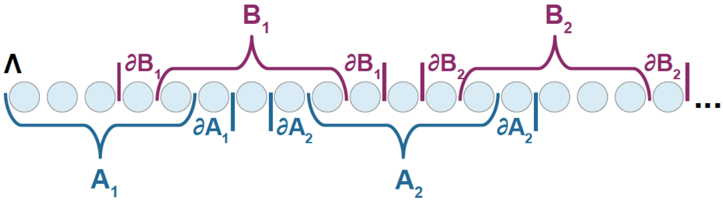


Quasi-factorization of the relative entropy

STEP 1



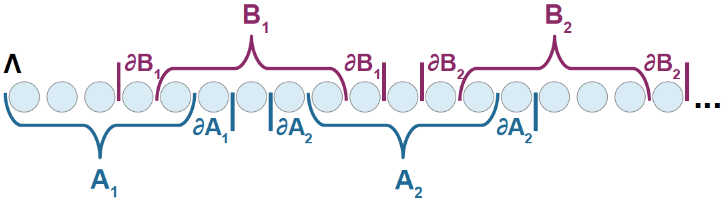
$$A = \bigcup_{i=1}^{n} A_i \text{ and } B = \bigcup_{j=1}^{n} B_j$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \frac{1}{1 - 2 \| h(\sigma_{A^c B^c}) \|_\infty} \left[D_A(\rho_\Lambda || \sigma_\Lambda) + D_B(\rho_\Lambda || \sigma_\Lambda) \right],$$

$$h(\sigma_{A^c B^c}) := \sigma_{A^c}^{-1/2} \otimes \sigma_{B^c}^{-1/2} \sigma_{A^c B^c} \sigma_{A^c}^{-1/2} \otimes \sigma_{B^c}^{-1/2} - \mathbb{1}_{A^c B^c}.$$

Quasi-factorization of the relative entropy

STEP 1



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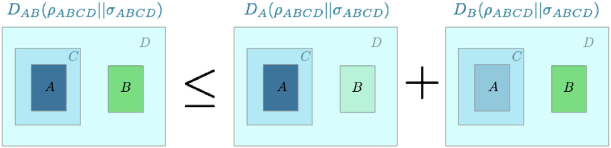
Quasi-factorization for QMC **(Heat-bath)**

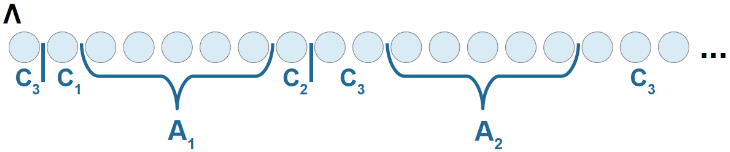
Let $\mathcal{H}_{ABCD} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \otimes \mathcal{H}_D$, where system C shields A from BD and $\rho_{ABCD}, \sigma_{ABCD} \in \mathcal{S}_{ABCD}$, such that σ_{ABCD} is a quantum Markov chain between $A \leftrightarrow C \leftrightarrow BD$. Then, the following holds

$$D_{AB}(\rho_{ABCD}||\sigma_{ABCD}) \leq \left[D_A(\rho_{ABCD}||\sigma_{ABCD}) + D_B(\rho_{ABCD}||\sigma_{ABCD}) \right].$$



Sketch of the proof

STEP 2



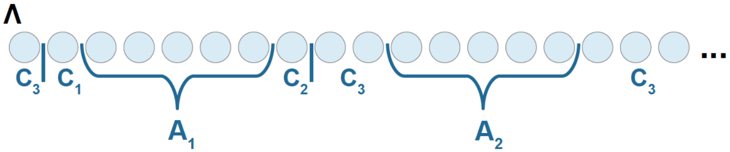
$$D_A(\rho_\Lambda || \sigma_\Lambda) \leq \sum_{i=1}^{n} D_{A_i}(\rho_\Lambda || \sigma_\Lambda)$$

σ_Λ is a QMC between $A_1 \leftrightarrow \partial A_1 \leftrightarrow \Lambda \setminus (A_1 \cup \partial A_1)$

$$\sigma_\Lambda = \bigoplus_{i \in I} \sigma_{A_1 (\partial a_1)_i^L} \otimes \sigma_{(\partial a_1)_i^R \Lambda \setminus (A_1 \cup \partial A_1)}$$

Sketch of the proof

STEP 2



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Heat-bath dynamics in 1D

Assumption 1

In a tripartite Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_C \otimes \mathcal{H}_B$, A and B not connected, we have

$$\|h(\sigma_{AB})\|_\infty = \left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty \leq K < \frac{1}{2}.$$

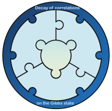
In particular, Gibbs states at high enough temperature satisfy this.

Assumption 2

For any $B \subset \Lambda$, $B = B_1 \cup B_2$, it holds:

$$D_B(\rho_\Lambda || \sigma_\Lambda) \leq f(\sigma_{B\partial}) \left(D_{B_1}(\rho_\Lambda || \sigma_\Lambda) + D_{B_2}(\rho_\Lambda || \sigma_\Lambda) \right).$$

In particular, tensor products satisfy this (with $f = 1$).



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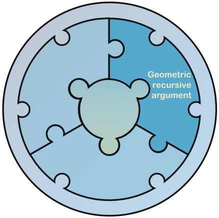
In particular, tensor products satisfy this (with $f = 1$).

Heat-bath dynamics in 1D

STEP 3

$$\text{Assumption 1} \Rightarrow \alpha(\mathcal{L}_\Lambda^*) \geq \tilde{K} \min_{i \in \{1, \ldots n\}} \left\{ \alpha_\Lambda(\mathcal{L}_{A_i}^*), \alpha_\Lambda(\mathcal{L}_{B_i}^*) \right\}$$



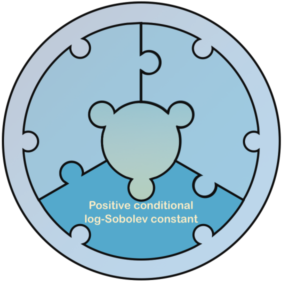
Using locality of the Lindbladian

$$\mathcal{L}_A^* + \mathcal{L}_B^* = \mathcal{L}_{A \cup B}^* + \mathcal{L}_{A \cap B}^*.$$

Heat-bath dynamics in 1D

STEP 4

$$\text{Assumption 2} \Rightarrow \alpha_\Lambda(\mathcal{L}^*_{A_i}) \geq g(\sigma_{A_i\partial}) > 0.$$



Heat-bath dynamics in 1D

Theorem (Heat-bath)

In 1D, if Assumptions 1 and 2 hold, for a k-local commuting Hamiltonian, the heat-bath dynamics has a positive log-Sobolev constant.

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