When does a quantum dissipative evolution mix rapidly?

Rapid mixing for 1D Davies generators and applications

Ángela Capel Cuevas
(Universität Tübingen)

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5 July 2022

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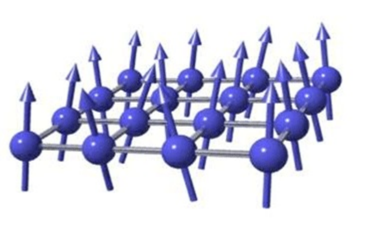

Notation: An open quantum many-body system

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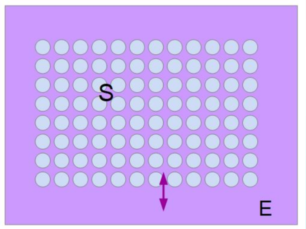
No experiment can be executed at zero temperature or be completely shielded from noise.



- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$.
- Density matrices: $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \, : \, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

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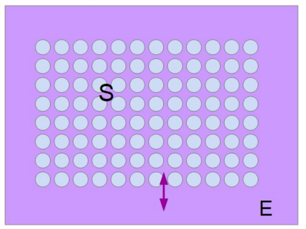

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Quantum Markov semigroups / Dissipative quantum evolution

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The infinitesimal generator $\mathcal{L}_\Lambda^*$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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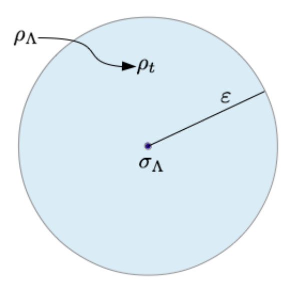

Dissipative quantum systems

Mixing $\Leftrightarrow$ Convergence

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We assume that $\{\mathcal{T}_t^*\}_{t \geq 0}$ has a unique full-rank invariant state which we denote by σ_Λ.

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We also assume that the quantum Markov process studied is **reversible**, i.e., satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}_\Lambda(g)\rangle_\sigma = \langle \mathcal{L}_\Lambda(f), g\rangle_\sigma,$$

for every $f, g \in \mathcal{B}_\Lambda$ and Hermitian, where

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Mixing time

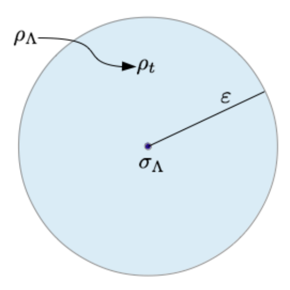
- Under the previous conditions, there is always convergence to σ_Λ.
- How fast does convergence happen?

Note $\mathcal{T}_\infty^*(\rho) := \sigma_\Lambda$ for every ρ.

Mixing time

We define the **mixing time** of $\{\mathcal{T}_t^*\}$ by

$$t_{\text{mix}}(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t^*(\rho) - \mathcal{T}_\infty^*(\rho) \right\|_1 \leq \varepsilon \right\}.$$



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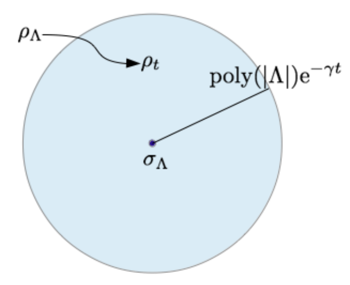

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Rapid mixing

We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|)e^{-\gamma t}.$$

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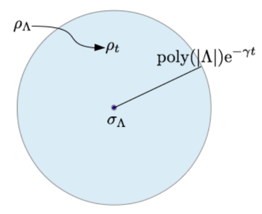

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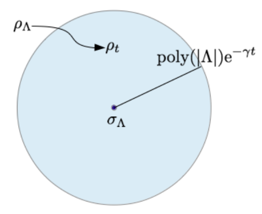

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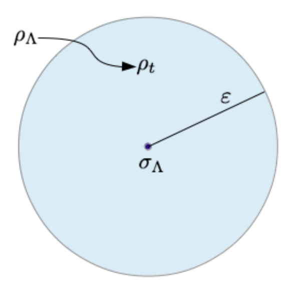
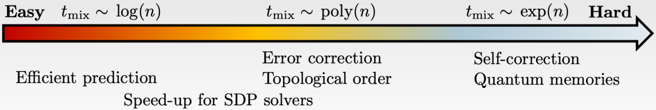


Quantum dissipative evolutions useful?

If **rapid mixing**, no **error correction:**



Further applications:

- Robust and efficient **preparation of topologically ordered phases** of matter via dissipation.
- **Classification of dissipative phases** of matter.
- Design of more efficient **quantum error-correcting codes** optimized for correlated Markovian noise models.

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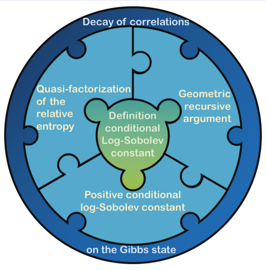
Quantum logarithmic Sobolev inequalities via quasi-factorization of the relative entropy.



Modified logarithmic Sobolev inequality (MLSI)

(in this talk, we simply call it **log-Sobolev inequality**)

Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

Master equation:

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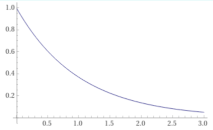

Lower bound for the derivative of $D(\rho_t || \sigma_\Lambda)$ in terms of itself:

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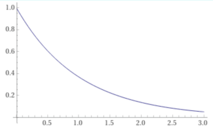

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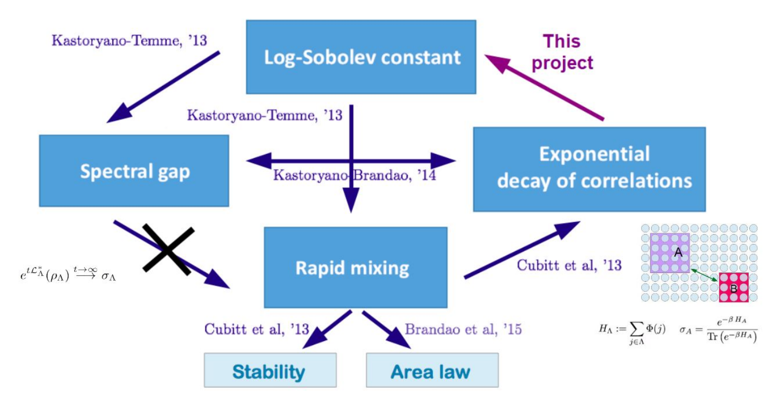
ANSWER: When our quantum dissipative evolution has a **positive MLSI**, it mixes rapidly.

Remark: And when it only has a positive spectral gap, in general it does not mix rapidly.

Quantum spin systems



Exp. decay of correlations:

$$\sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]| \leq K\, \mathrm{e}^{-\gamma d(A,B)} \quad .$$

Decay of correlations on Gibbs state

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A \cup B)^c} + H_{\partial A} + H_{\partial B}$,
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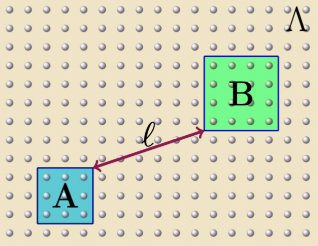


$\ell := \mathrm{dist}(A, B)$

Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta H_{A\cup B}} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \; ?$$

$$\mathrm{tr}_{A^c}[\sigma_\Lambda] \otimes \mathrm{tr}_{B^c}[\sigma_\Lambda] := \left(\sigma_\Lambda\right)_A \otimes \left(\sigma_\Lambda\right)_B \approx \mathrm{tr}_{(A\cup B)^c}[\sigma_\Lambda] := \left(\sigma_\Lambda\right)_{A\cup B} \; ?$$

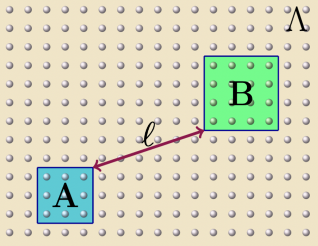
Decay of correlations on Gibbs state

Ángela Capel Cuevas
(Universität Tübingen)

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial B}$,
- **Gibbs state:** $\sigma_\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} / \operatorname{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.



$\ell := \operatorname{dist}(A, B)$

Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta H_{A\cup B}} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \ ?$$

$$\operatorname{tr}_{A^c}[\sigma_\Lambda] \otimes \operatorname{tr}_{B^c}[\sigma_\Lambda] := (\sigma_\Lambda)_A \otimes (\sigma_\Lambda)_B \approx \operatorname{tr}_{(A\cup B)^c}[\sigma_\Lambda] := (\sigma_\Lambda)_{A\cup B} \ ?$$

Decay of correlations on Gibbs state

3 different forms of **decay of correlations**.

Operator correlation

$$\text{Corr}_\sigma(A:B) := \sup_{\|O_A\|=\|O_B\|=1} |\text{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]|$$

Mutual information

$$I_\sigma(A:B) := D(\rho_{AB}||\rho_A \otimes \rho_B)$$

for $D(\rho\|\sigma) = \text{Tr}[\rho(\log\rho - \log\sigma)]$

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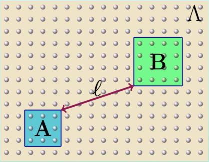
Mixing condition

$$\|h(\sigma_{AB})\|_\infty = \left\|\sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB}\right\|_\infty$$



Relation:

$$\frac{1}{2}\mathrm{Corr}_\sigma(A:B)^2 \leq I_\sigma(A:B)$$
$$\leq \left\|\sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB}\right\|_\infty .$$

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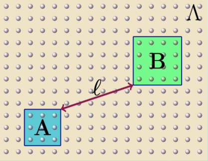

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Quantum spin systems

Thermalization

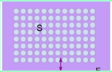
MLSI (log-Sobolev)

$$D(\rho_t||\sigma_\Lambda) \leq D(\rho_\Lambda||\sigma_\Lambda)e^{-2\,\alpha(\mathcal{L}^*_\Lambda)\,t}$$

Rapid Mixing

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|)e^{-\gamma t}$$

$$e^{t\mathcal{L}^*_\Lambda}(\rho_\Lambda) \xrightarrow{t\to\infty} \sigma_\Lambda$$



Spectral gap

$$\text{Var}(\rho_t, \sigma_\Lambda) \leq \text{Var}(\rho_\Lambda, \sigma_\Lambda)e^{-\lambda(\mathcal{L}^*_\Lambda)\,t}$$

Decay of correlations

Mixing condition

$$\left\| \sigma_A^{-1} \otimes \sigma_B^{-1}\sigma_{AB} - \mathbb{1}_{AB} \right\|_\infty \leq K e^{-\gamma\, d(A,B)}$$

Mutual information

$$I_\sigma(A:B) \leq K e^{-\gamma\, d(A,B)}$$

Operator correlation

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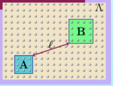
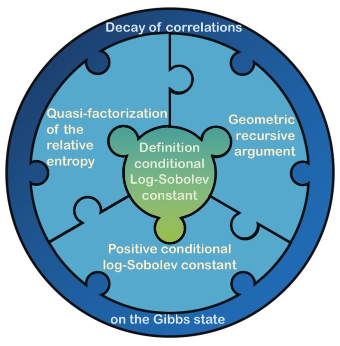


When does a quantum dissipative evolution have a **positive MLSI**?



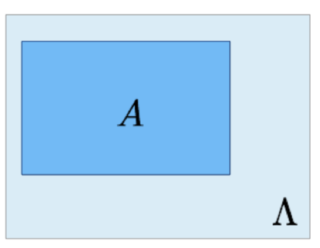
Objective

MLSI constant

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|\Lambda|) > 0.$$



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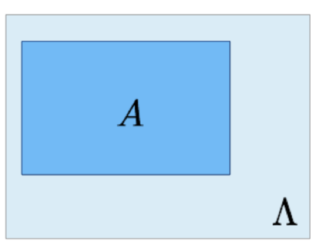

Can we prove something like

$$\alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|A|)\, \alpha(\mathcal{L}_A^*) > 0 \ ?$$

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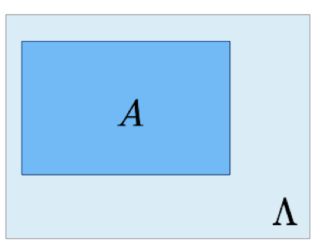

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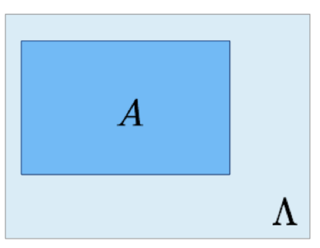

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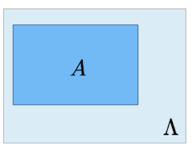
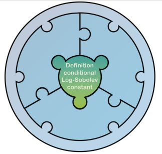
Conditional MLSI constant





MLSI constant

The **MLSI constant** of $\mathcal{L}_\Lambda^* = \sum_{k \in \Lambda} \mathcal{L}_k^*$ is defined by

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

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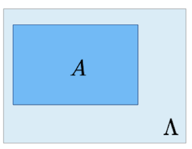

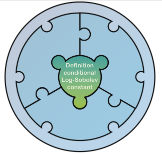

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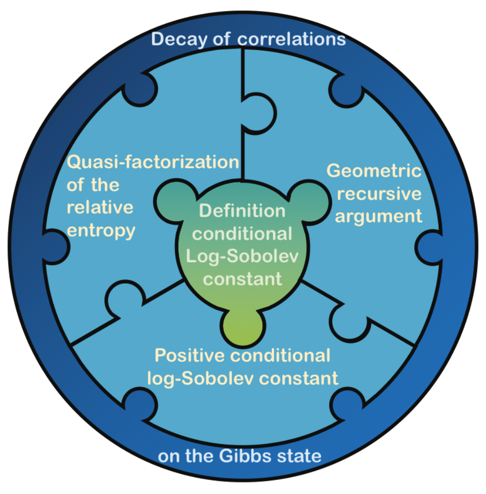
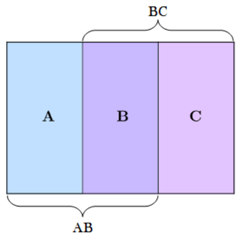
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

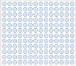
$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Example: Tensor product fixed point

(C.-Lucia-Pérez García '18) $\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x\in\Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$ **heat-bath**

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda\|\sigma_\Lambda) - D(\rho_{x^c}\|\sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x ,$$

$$D(\rho_\Lambda\|\sigma_\Lambda) \leq$$

$$\leq \sum_{x\in\Lambda} D_x(\rho_\Lambda\|\sigma_\Lambda)$$

$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda\in\mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2D_x(\rho_\Lambda\|\sigma_\Lambda)}$

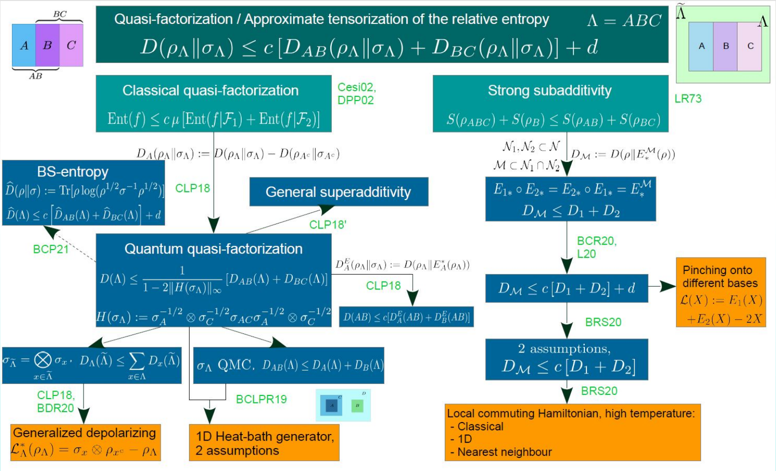
$$\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x^*)}$$

$$\leq \frac{1}{2\inf_{x\in\Lambda}\alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x\in\Lambda} -\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]$$

$$= \frac{1}{2\inf_{x\in\Lambda}\alpha_\Lambda(\mathcal{L}_x^*)} \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right)$$

$$\leq \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right).$$

Quasi-factorization of the relative entropy



Most recent result

MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '21)

Let $\mathcal{L}_\Lambda^{D;*}$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^{D;*}$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^{D;*}) = \Omega(\ln(|\Lambda|)^{-1})$.

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$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

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For $\alpha(\mathcal{L}_\Lambda^*)$ a **MLSI constant**:

$$\|\rho_t - \sigma_\Lambda\|_1 \leq \sqrt{2 \log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\, t}.$$

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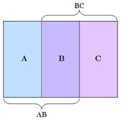
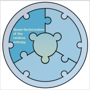
Sketch of the proof





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Heat-bath cond. expectation: $E_A^*(\cdot) := \lim_{n \to \infty} \left(\sigma_\Lambda^{1/2} \sigma_{A^c}^{-1/2} \operatorname{tr}_A[\,\cdot\,]\, \sigma_{A^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n$

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Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

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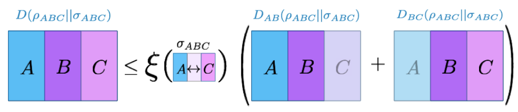

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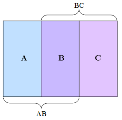

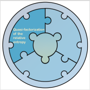

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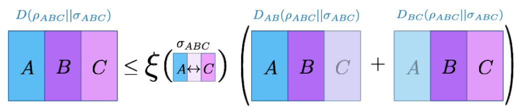

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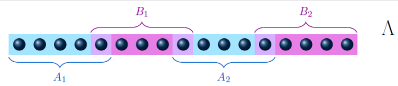
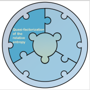
(Universität Tübingen)

Sketch of the proof: Quasi-factorization





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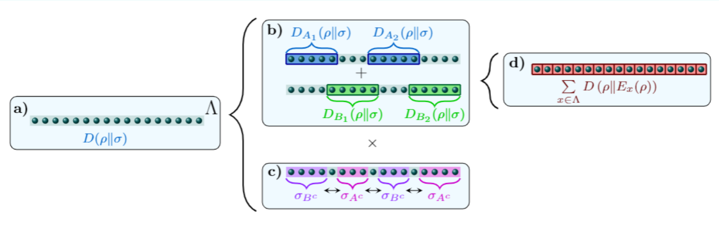

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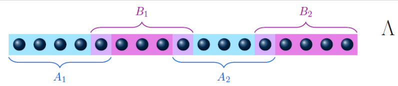

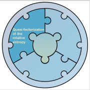

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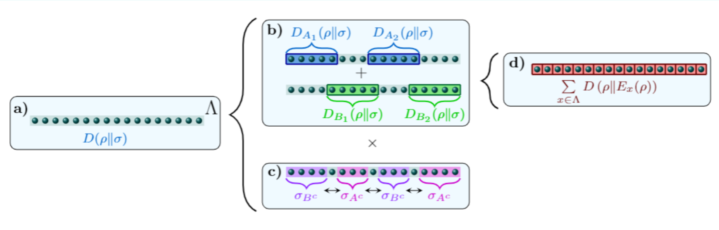

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Thank you for your attention!

Do you have any questions?

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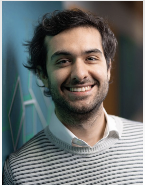
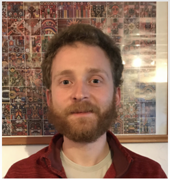
U. Complutense Madrid

Angelo Lucia
U. Complutense Madrid



Cambyse Rouzé
T. U. Munich



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Inria Paris

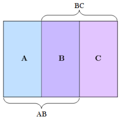
Daniel Stilck Franca
ENS Lyon

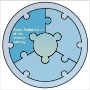
Antonio Pérez-Hernández
UNED Madrid

Andreas Bluhm
U. Copenhagen

Li Gao
U. Houston

Proof: Conditional relative entropies + Quasi-factorization





Conditional relative entropies: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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Heat-bath cond. expectation: $E_A^*(\cdot) := \lim_{n \to \infty} \left(\sigma_\Lambda^{1/2} \sigma_{A^c}^{-1/2} \operatorname{tr}_A[\,\cdot\,]\, \sigma_{A^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n$.

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Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

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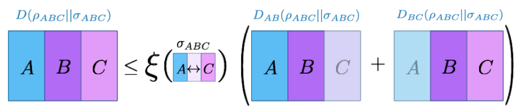
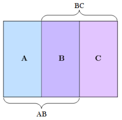
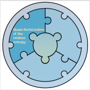


Proof: Conditional relative entropies + Quasi-factorization





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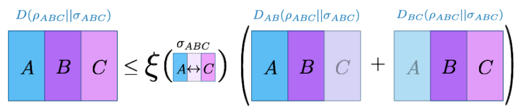

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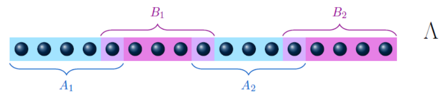

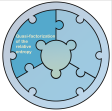

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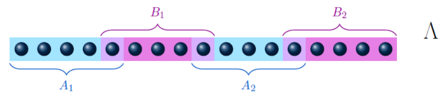

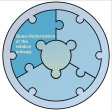

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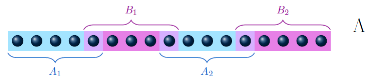

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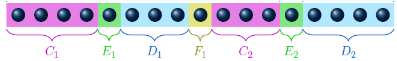

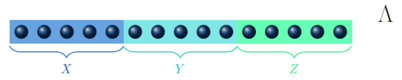

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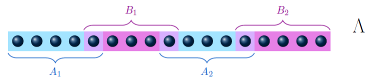

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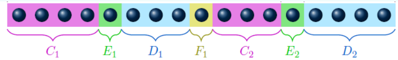

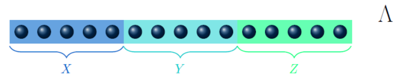

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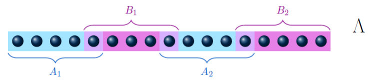

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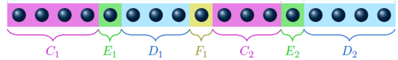

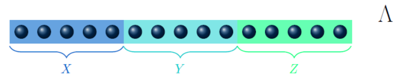

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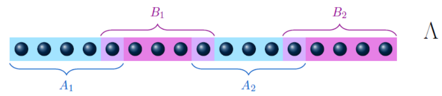
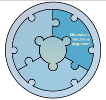
Proof: Geometric recursive argument

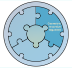




Let us recall: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
$D_A^E(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| E_A^*(\rho_\Lambda))$.

Comparison conditional rel. ent. (Bardet-C.-Rouzé, '20)

$$D_A(\rho_\Lambda \| \sigma_\Lambda) \leq D_A^E(\rho_\Lambda \| \sigma_\Lambda)$$

Therefore, by this and  +  , we have:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{A^c B^c}) \sum_i \left[D_{A_i}^E(\rho_\Lambda \| \sigma_\Lambda) + D_{B_i}^E(\rho_\Lambda \| \sigma_\Lambda) \right],$$

and thus $\alpha(\mathcal{L}_\Lambda^{H;*}) \geq \frac{K}{\xi(\sigma_{A^c B^c})} \min \left\{ \alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}), \alpha_{B_i}(\mathcal{L}_\Lambda^{H;*}) \right\},$

for

$$\alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}) = \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}\left[\mathcal{L}_{A_i}^{H;*}(\rho_\Lambda)(\ln \rho_\Lambda - \ln \sigma_\Lambda) \right]}{D(\rho_\Lambda \| E_{A_i}^*(\rho_\Lambda))} .$$

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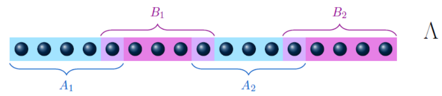

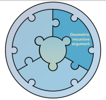

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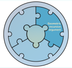
Therefore, by this and  +  , we have:

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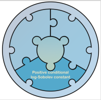
Proof: Positive CMLSI



Reduction of conditional relative entropies (Gao-Rouzé, '21)

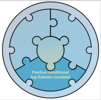
$$D(\rho_\Lambda \| E^*_{A_i}(\rho_\Lambda)) \leq 4 k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E^*_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda},$$

where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

Proof: Positive CMLSI



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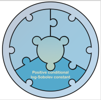

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CMLSI (Gao-Rouzé, '21)

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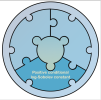

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CMLSI (Gao-Rouzé, '21)

Last step

Heat-bath cond. expectation:
$E_A^{H;*}(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2}\sigma_{A^c}^{-1/2}\,\mathrm{tr}_A[\,\cdot\,]\,\sigma_{A^c}^{-1/2}\sigma_\Lambda^{1/2}\right)^n \, .$
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Davies and heat-bath dynamics (Bardet-C.-Rouzé, '20)

The conditional expectations associated to Davies and heat-bath dynamics coincide.

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For $\mathcal{L}_\Lambda^{D;*}$, there is a positive MLSI constant $\alpha(\mathcal{L}_\Lambda^{D;*}) = \Omega(\ln|\Lambda|^{-1})$. Therefore, $\mathcal{L}_\Lambda^{D;*}$ has rapid mixing.

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