Functional inequalities in non-commutative $\mathbb{L}_p$ spaces for quantum many-body systems

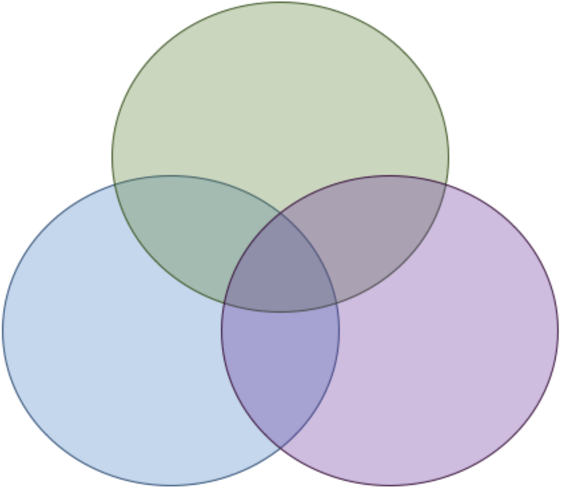
Ángela Capel (Technische Universität München)

International Workshop on Operator Theory and its Applications, 10 August 2021

Technical University of Munich

Munich Center for Quantum Science and Technology

Field of study



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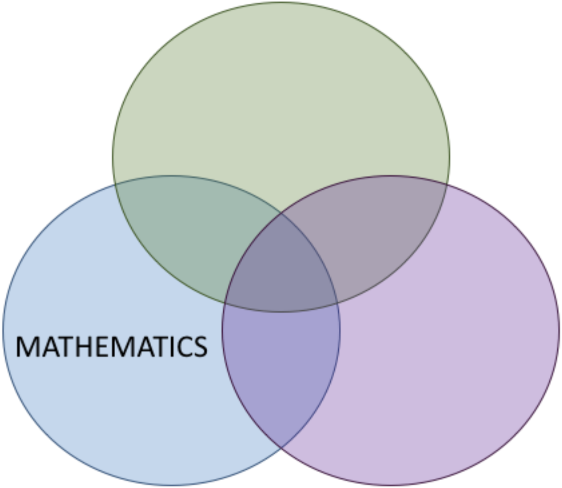

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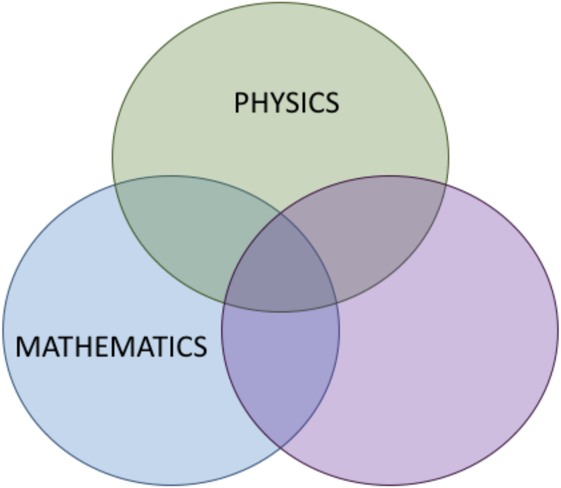

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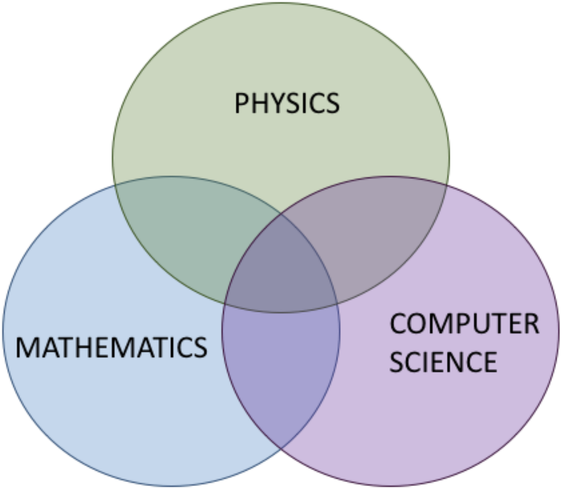

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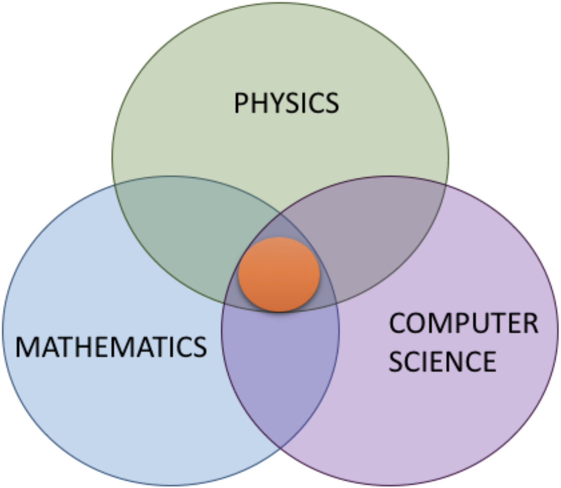

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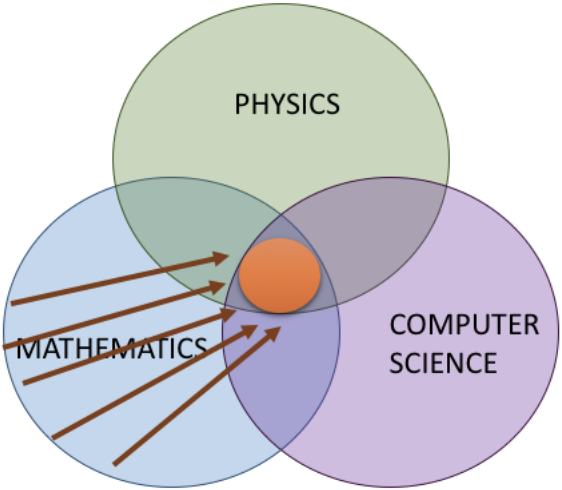

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Q. information theory $\longleftrightarrow$ Q. many-body physics

Communication channels $\longleftrightarrow$ Physical interactions

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Dissipative evolutions of quantum many-body systems

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Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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Contents

1.1 Quantum dissipative systems

Open quantum systems

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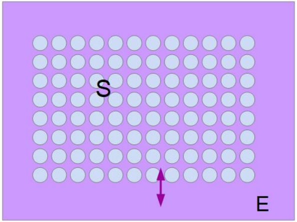
$\Rightarrow$ Open quantum many-body systems.



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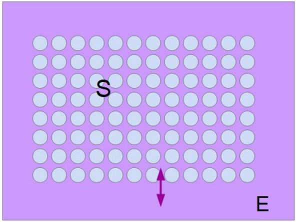

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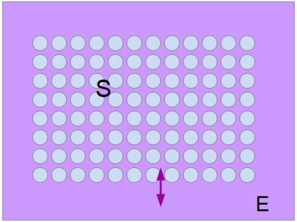

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Postulates of quantum mechanics

Postulate 1

Given an isolated physical system, there is a complex Hilbert space $\mathcal{H}$ associated to it, which is known as the **state space** of the system.

Moreover, the physical system is completely described by its **state vector**, which is a unitary vector in the state space.

Postulate 2

Given an isolated physical system, its evolution is described by a **unitary transformation** in the Hilbert space.

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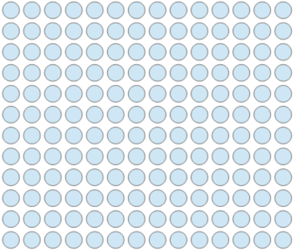

Figure: A quantum spin lattice system.

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- To every site $x \in \Lambda$ we associate $\mathcal{H}_x$ $(= \mathbb{C}^D)$.
- The global Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x\in\Lambda} \mathcal{H}_x$.
- The set of bounded linear endomorphisms on $\mathcal{H}_\Lambda$ is denoted by $\mathcal{B}_\Lambda := \mathcal{B}(\mathcal{H}_\Lambda)$.
- The set of density matrices is denoted by $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \, : \, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

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Physical evolution: $\rho \mapsto U\rho U^* \rightsquigarrow$ Reversible

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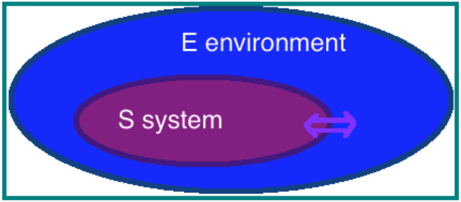
Open systems $\Rightarrow$ Environment and system interact.



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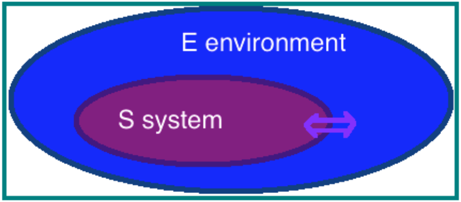

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State for the environment: $|\psi\rangle\langle\psi|_E$

$$\rho \mapsto \rho \otimes |\psi\rangle\langle\psi|_E \mapsto U\left(\rho \otimes |\psi\rangle\langle\psi|_E\right)U^* \mapsto \mathrm{tr}_E[U\left(\rho \otimes |\psi\rangle\langle\psi|_E\right)U^*] = \tilde{\rho}$$

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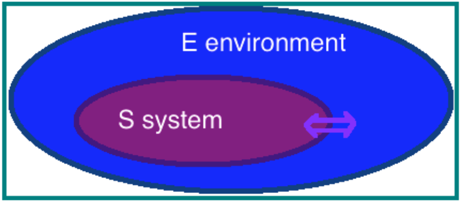

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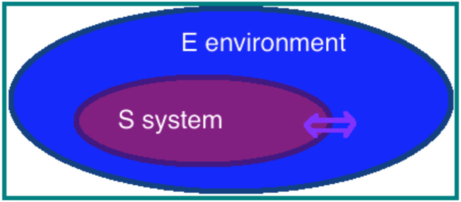

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The infinitesimal generator $\mathcal{L}_\Lambda^*$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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We assume that $\{\mathcal{T}_t^*\}_{t\geq 0}$ has a unique full-rank invariant state, which we denote by σ.

Reversibility

We also assume that the quantum Markov process studied is **reversible**, i.e., satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}(g) \rangle_\sigma = \langle \mathcal{L}(f), g \rangle_\sigma$$

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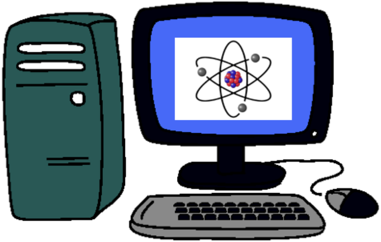
Main objective:



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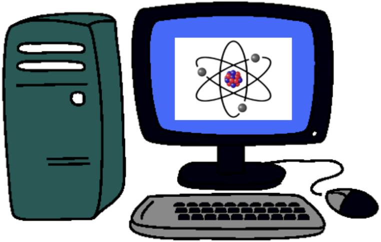

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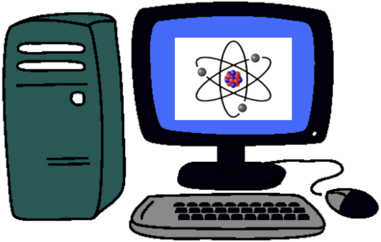

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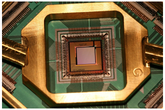

Recent change of perspective ⇒ Resource to exploit

New area:

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to create artificial evolutions in which the dissipative process works in favor (protecting the system from noisy evolutions).

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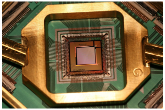

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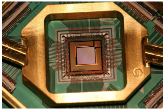

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Mixing time

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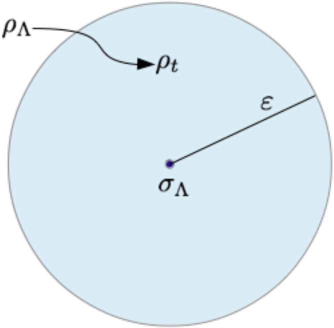
We define the **mixing time** of $\{\mathcal{T}_t^*\}$ by

$$\tau(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t^*(\rho) - \mathcal{T}_\infty^*(\rho) \right\|_1 \leq \varepsilon \right\}.$$



Rapid mixing

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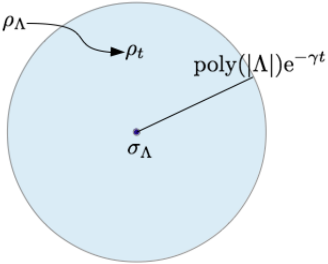
We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$



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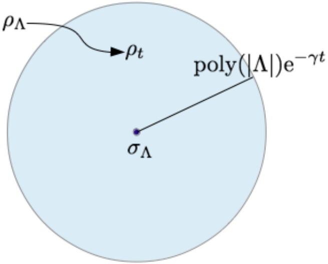

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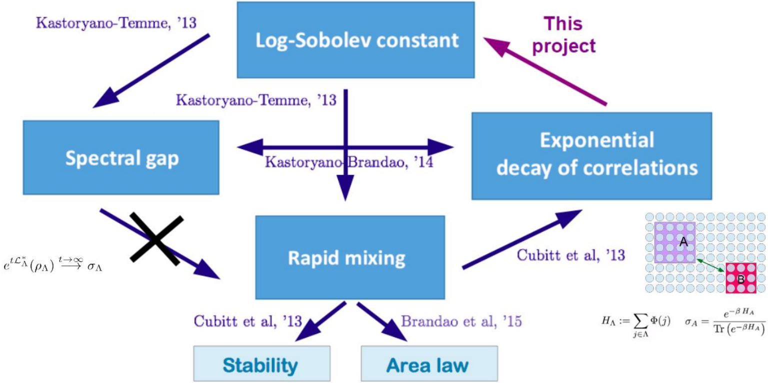
1.2 Logarithmic Sobolev inequalities

Quantum spin systems



Log-Sobolev inequality (MLSI)

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$$\partial_t D(\rho_t||\sigma_\Lambda) = \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)].$$

Lower bound for the derivative of $D(\rho_t||\sigma_\Lambda)$ in terms of itself:

$$2\alpha D(\rho_t||\sigma_\Lambda) \leq -\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)].$$

Log-Sobolev inequality (MLSI)

Recall: $\rho_t := \mathcal{T}_t^*(\rho)$.

Liouville's equation:

$$\partial_t \rho_t = \mathcal{L}_\Lambda^*(\rho_t).$$

Relative entropy of ρ_t and σ_Λ:

$$D(\rho_t||\sigma_\Lambda) = \operatorname{tr}[\rho_t(\log \rho_t - \log \sigma_\Lambda)].$$

Differentiating:

$$\partial_t D(\rho_t||\sigma_\Lambda) = \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)].$$

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Log-Sobolev constant

Log-Sobolev constant

The **log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ is defined as:

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

If $\alpha(\mathcal{L}_\Lambda^*) > 0$:

$$D(\rho_t || \sigma_\Lambda) \leq D(\rho_\Lambda || \sigma_\Lambda) e^{-2\,\alpha(\mathcal{L}_\Lambda^*)\,t},$$

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and with **Pinsker's inequality**, we have:

$$\left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \sqrt{2D(\rho_\Lambda || \sigma_\Lambda)}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t} \leq \sqrt{2 \log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\,t}.$$

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Log-Sobolev constant ⇒ Rapid mixing.

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Log-Sobolev constant $\Rightarrow$ Rapid mixing.

Problem

Find positive log-Sobolev constants!

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Find positive log-Sobolev constants!

First main objective of this project

Develop a strategy to find positive log Sobolev constants from static properties on the fixed point.

Second main objective of this project

Apply that strategy to certain dissipative dynamics.

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2 Results

2.1 Strategy

Classical spin systems

(Cesi, Dai Pra-Paganoni-Posta, '02)

(1) Quasi-factorization of the entropy (in terms of a conditional entropy).

+

(2) Recursive geometric argument.
Lower bound for the global log-Sobolev constant in terms of the log-Sobolev constant of a size-fixed region.

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⇓

Positive log-Sobolev constant.

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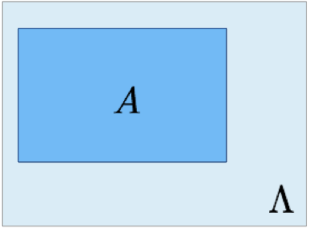
$\Downarrow$

Positive log-Sobolev constant.

Objective

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|\Lambda|) > 0.$$



Can we prove something like

$$\alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|A|)\, \alpha(\mathcal{L}_A^*) > 0 \ ?$$

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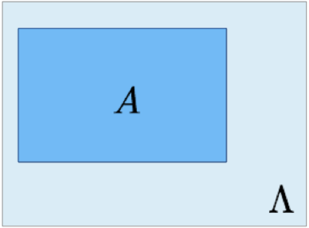

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No, but we can prove

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Conditional log-Sobolev constant

Log-Sobolev constant

Let $\mathcal{L}_\Lambda^* : \mathcal{S}_\Lambda \to \mathcal{S}_\Lambda$ be a primitive reversible Lindbladian with stationary state σ_Λ. We define the **log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ by

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

Conditional log-Sobolev constant

Let $\mathcal{L}_\Lambda^* : \mathcal{S}_\Lambda \to \mathcal{S}_\Lambda$ be a primitive reversible Lindbladian with stationary state σ_Λ, $A \subseteq \Lambda$. We define the **conditional log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ on A by

$$\alpha_\Lambda(\mathcal{L}_A^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_A^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D_A(\rho_\Lambda || \sigma_\Lambda)}$$

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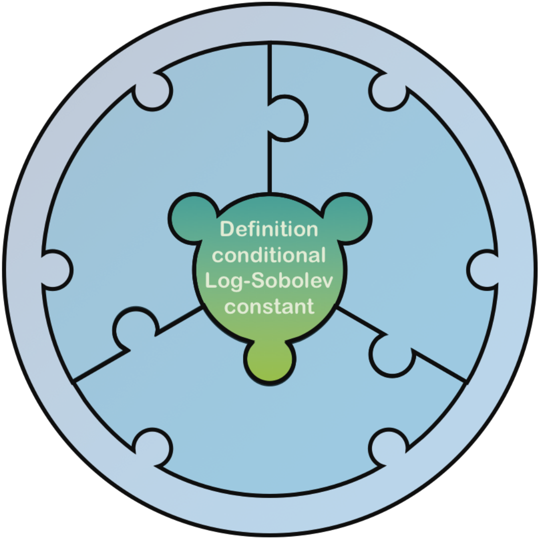
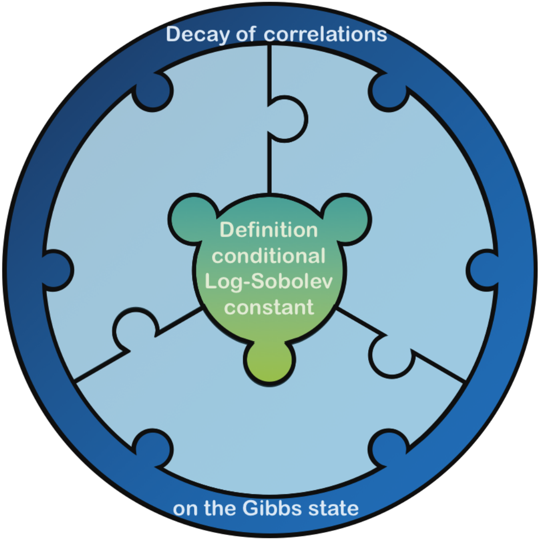
Strategy



Strategy


Decay of correlations
Definition
conditional
Log-Sobolev
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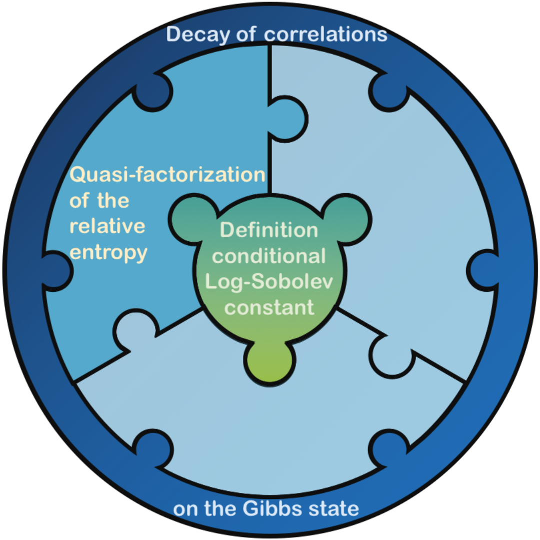
on the Gibbs state

Strategy


Decay of correlations
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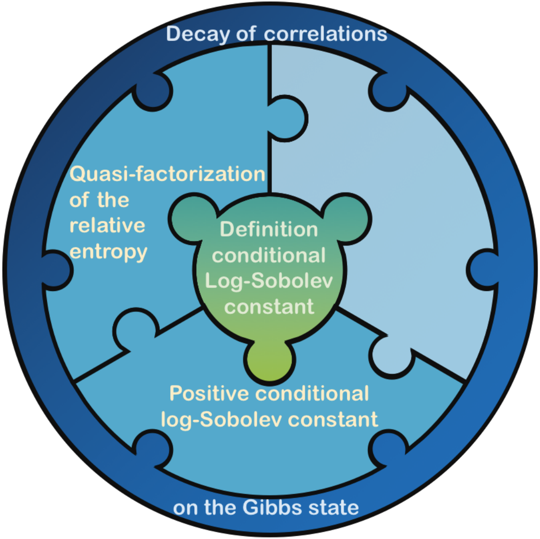
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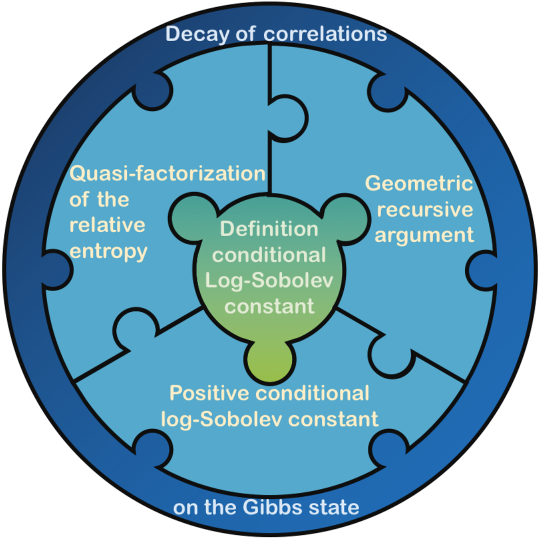
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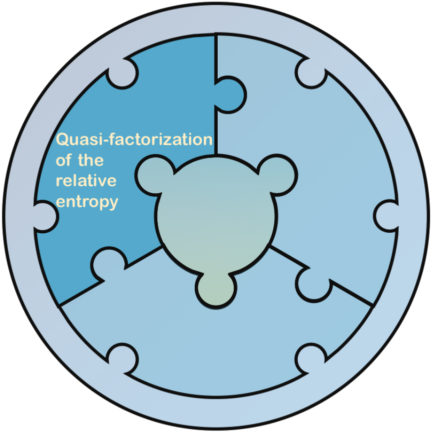
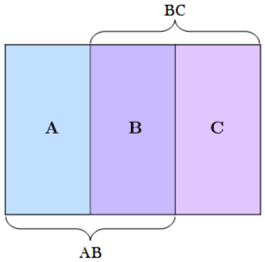
2.2 Part 2: Quasi-factorization of the relative entropy



Statement of the problem



Problem

Let $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. Can we prove something like

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right] \ ?$$

Quantum relative entropy

$$D(\rho||\sigma) = \operatorname{tr}\left[\rho(\log\rho - \log\sigma)\right]$$

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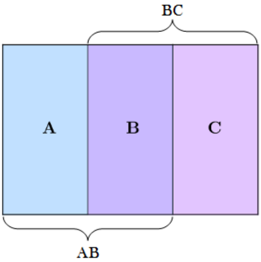

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Classical case, Dai Pra et al. '02

$$\mathrm{Ent}_\mu(f) \leq \frac{1}{1 - 4\|h-1\|_\infty} \, \mu \left[\mathrm{Ent}_\mu(f \mid \mathcal{F}_1) + \mathrm{Ent}_\mu(f \mid \mathcal{F}_2) \right],$$

where $h = \dfrac{d\mu}{d\bar{\mu}}$.

Problem

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right]$$

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where $h = \dfrac{d\mu}{d\bar{\mu}}$.

Classical entropy and conditional entropy

Entropy:

$$\text{Ent}_\mu(f) = \mu(f \log f) - \mu(f) \log \mu(f).$$

Conditional entropy:

$$\text{Ent}_\mu(f \mid \mathcal{G}) = \mu(f \log f \mid \mathcal{G}) - \mu(f \mid \mathcal{G}) \log \mu(f \mid \mathcal{G}).$$

Problem

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right]$$

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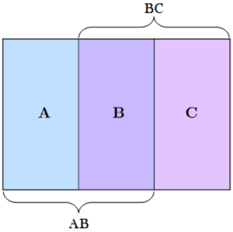


Figure: Choice of indices in $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$.

Result of **quasi-factorization** of the relative entropy, for every $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$:

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Conditional Relative Entropy (C.-Lucia-Pérez García '18)

The conditional relative entropy is given by:

$$D_A(\rho_{AB}||\sigma_{AB}) = D(\rho_{AB}||\sigma_{AB}) - D(\rho_B||\sigma_B)$$

for every $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$.

Quasi-factorization for the CRE (C.-Lucia-Pérez García '18)

Let $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. Then, the following inequality holds

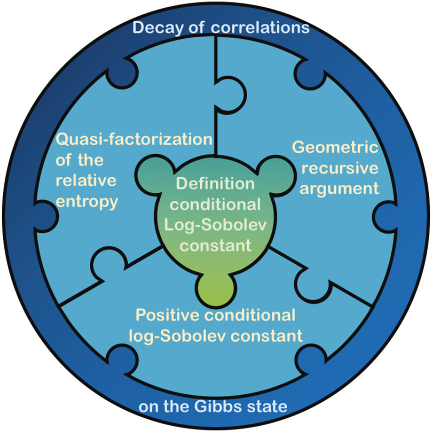
$$D(\rho_{ABC}||\sigma_{ABC}) \leq \frac{1}{1 - 2\|H(\sigma_{AC})\|_\infty} \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where

$$H(\sigma_{AC}) = \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \, \sigma_{AC} \, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC}.$$

Note that $H(\sigma_{AC}) = 0$ if σ_{AC} is a tensor product between A and C.

2.3 Part 3: Log-Sobolev constants



Quantum spin lattices

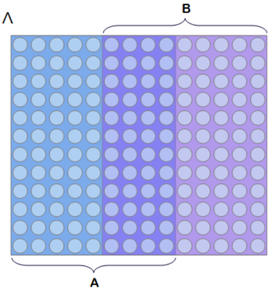


Figure: A quantum spin lattice system Λ and $A, B \subseteq \Lambda$ such that $A \cup B = \Lambda$.

Problem

For a certain $\mathcal{L}_\Lambda^*$, can we prove $\alpha(\mathcal{L}_\Lambda^*) > 0$ using the result of quasi-factorization of the relative entropy?

Heat-bath with tensor product fixed point

Theorem

The **heat-bath dynamics**, with tensor product fixed point, has a positive log-Sobolev constant.

Consider the local and global Lindbladians

$$\mathcal{L}_x^* := \mathbb{E}_x^* - \mathbb{1}_\Lambda, \; \mathcal{L}_\Lambda^* = \sum_{x \in \Lambda} \mathcal{L}_x^*$$

Since

$$\mathbb{E}_x^*(\rho_\Lambda) = \sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} = \sigma_x \otimes \rho_{x^c}$$

for every $\rho_\Lambda \in \mathcal{S}_\Lambda$, we have

$$\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda).$$

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for every $\rho_\Lambda \in \mathcal{S}_\Lambda$, we have

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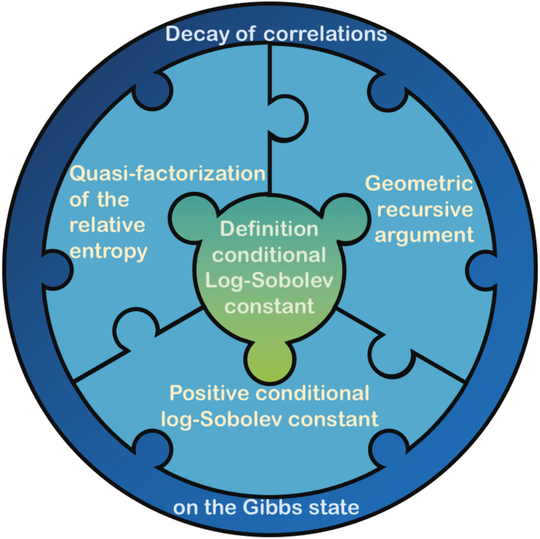
Strategy


Decay of correlations
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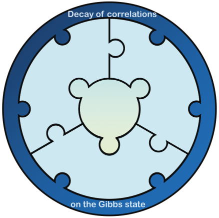
Heat-bath with tensor product fixed point

Assumption

$$\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x .$$



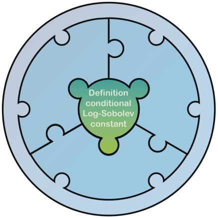
Heat-bath with tensor product fixed point

Conditional log-Sobolev constant

For $x \in \Lambda$, we define the **conditional log-Sobolev constant** of $\mathcal{L}_\Lambda^*$ in x by

$$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 D_x(\rho_\Lambda || \sigma_\Lambda)},$$

where σ_Λ is the fixed point of the evolution, and $D_x(\rho_\Lambda || \sigma_\Lambda)$ is the conditional relative entropy.

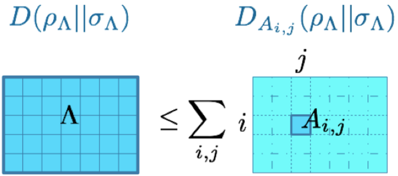


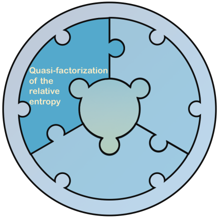
Heat-bath with tensor product fixed point

General quasi-factorization for σ a tensor product

Let $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$ such that $\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x$. The following inequality holds:

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda).$$

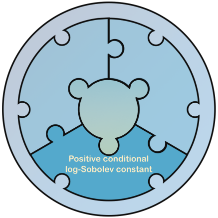




Heat-bath with tensor product fixed point

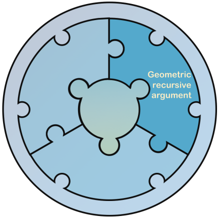
Lemma (Positivity of the conditional log-Sobolev constant)

$$\alpha_\Lambda(\mathcal{L}_x^*) \geq \frac{1}{2}.$$



Heat-bath with tensor product fixed point

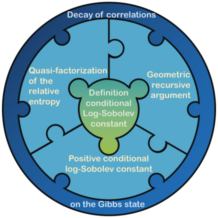
$$
\begin{aligned}
D(\rho_\Lambda||\sigma_\Lambda) &\leq \sum_{x\in\Lambda} D_x(\rho_\Lambda||\sigma_\Lambda) \\
&\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x^*)} \\
&\leq \frac{1}{2\inf\limits_{x\in\Lambda}\alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x\in\Lambda} -\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)] \\
&= \frac{1}{2\inf\limits_{x\in\Lambda}\alpha_\Lambda(\mathcal{L}_x^*)} \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right) \\
&\leq \left(-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right).
\end{aligned}
$$



Heat-bath with tensor product fixed point

Positive log-Sobolev constant

$$\alpha(\mathcal{L}_\Lambda^*) \geq \frac{1}{2}.$$



Conclusions

In this talk, we have:

- Introduced some notions on quantum dissipative evolutions and logarithmic Sobolev inequalities.
- Presented a strategy to prove positivity for logarithmic Sobolev inequalities.
- Applied that strategy to a particular setting.

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