Entropy decay for Davies semigroups of a one dimensional quantum lattice

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Technical University of Munich

Munich Center for Quantum Science and Technology

Open quantum systems

Problem

Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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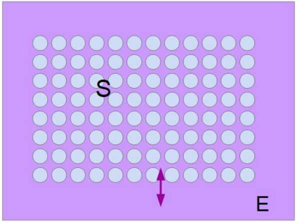

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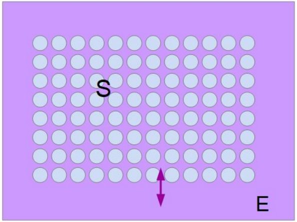

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- The continuous-time evolution of a state on S is given by a q. Markov semigroup (**Markovian approximation**).

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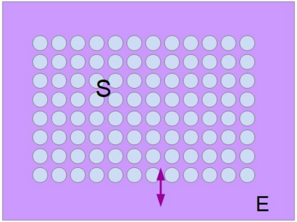

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A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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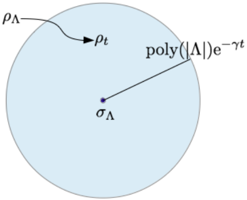
Rapid mixing

We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t} .$$



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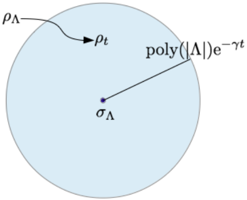

Modified logarithmic Sobolev inequality

Relative entropy: $D(\rho||\sigma) := \operatorname{tr}[\rho(\log \rho - \log \sigma)]$

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If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) > 0$:

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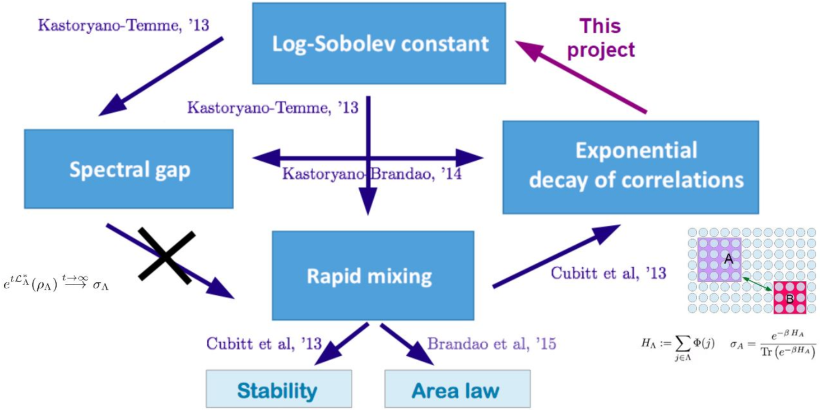
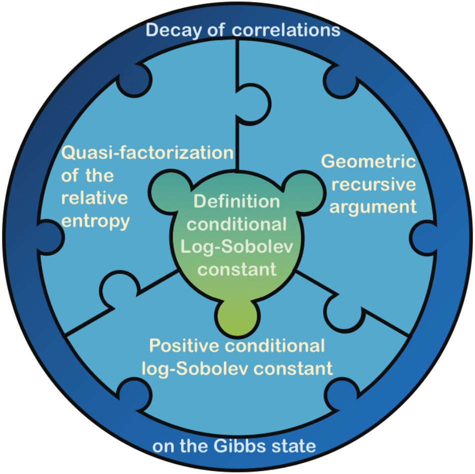
QUANTUM SPIN SYSTEMS

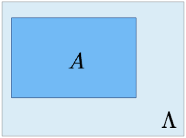


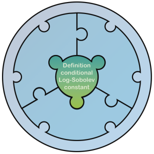
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Conditional MLSI constant





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The **conditional MLSI constant** of $\mathcal{L}_\Lambda^*$ on $A \subset \Lambda$ is defined by

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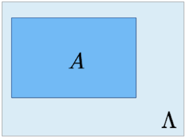

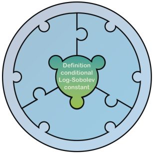

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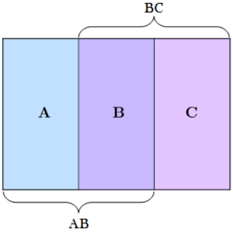
QUASI-FACTORIZATION OF THE RELATIVE ENTROPY



QUASI-FACTORIZATION OF THE RELATIVE ENTROPY

Given $\Lambda = ABC$, it is an inequality of the form:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

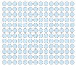
for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Example: Tensor product fixed point

(C.-Lucia-Pérez García '18)

$$\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sum_{x \in \Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$$

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x \in \Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

$$\leq \sum_{x \in \Lambda} D_x(\rho_\Lambda || \sigma_\Lambda)$$

$$\alpha_\Lambda(\mathcal{L}_x^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 D_x(\rho_\Lambda || \sigma_\Lambda)}$$

$$\leq \sum_{x \in \Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 \alpha_\Lambda(\mathcal{L}_x^*)}$$

$$\leq \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \sum_{x \in \Lambda} - \operatorname{tr}[\mathcal{L}_x^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]$$

$$= \frac{1}{2 \inf\limits_{x \in \Lambda} \alpha_\Lambda(\mathcal{L}_x^*)} \left(- \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \right)$$

$$\leq \left(- \operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)] \right) .$$

Dynamics

Let $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\text{tr}\left[e^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

$$\mathcal{L}_\Lambda^{H;*}(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right)$$

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Davies generator

The **Davies generator** is given by:

$$\mathcal{L}_\Lambda^D(X) := i[H_\Lambda, X] + \sum_{x\in\Lambda} \mathcal{L}_x^D(X) \,,$$

where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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The **Schmidt generator** (Bravyi-Vyalyi '05) can be written as:

$$\mathcal{L}_\Lambda^S(X) = \sum_{x\in\Lambda} \left(E_x^S(X) - X \right),$$

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Previous results

Let us recall: For $\alpha(\mathcal{L}_\Lambda^*)$ a MLSI constant,

$$\|\rho_t - \sigma_\Lambda\|_1 \leq \sqrt{2\log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\, t}.$$

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Spectral gap for Davies and heat-bath (Kastoryano-Brandao, '16)

Let $\mathcal{L}_\Lambda^{H,D;*}$ be the **heat-bath** or **Davies** generator in 1D. Then, $\mathcal{L}_\Lambda^{H,D;*}$ has a positive spectral gap that is independent of the system size, for every temperature.

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Assumption 1

In a tripartite Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_C \otimes \mathcal{H}_B$, A and B not connected, we have

$$\left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty \leq K < \frac{1}{2}.$$

In particular, Gibbs states at high enough temperature satisfy this.

Assumption 2

For any $B \subset \Lambda$, $B = B_1 \cup B_2$, it holds:

$$D_B(\rho_\Lambda || \sigma_\Lambda) \leq f(\sigma_{B\partial}) \left(D_{B_1}(\rho_\Lambda || \sigma_\Lambda) + D_{B_2}(\rho_\Lambda || \sigma_\Lambda) \right).$$

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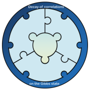

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Theorem (Bardet-C.-Lucia-Pérez García-Rouzé '19)

In 1D, if Assumptions 1 and 2 hold, for a k-local commuting Hamiltonian, the **heat-bath** dynamics has a positive log-Sobolev constant.

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For any $B \subset \Lambda$, $B = B_1 \cup B_2$, it holds:

$$D_B(\rho_\Lambda || \sigma_\Lambda) \leq f(\sigma_{B\partial}) \left(D_{B_1}(\rho_\Lambda || \sigma_\Lambda) + D_{B_2}(\rho_\Lambda || \sigma_\Lambda) \right).$$

In particular, tensor products satisfy this (with $f = 1$).

Theorem (Bardet-C.-Lucia-Pérez García-Rouzé '19)

In 1D, if Assumptions 1 and 2 hold, for a k-local commuting Hamiltonian, the **heat-bath** dynamics has a positive log-Sobolev constant.

Previous results

MLSI for Schmidt (C.-Rouzé-Stilck França '20)

Let H_Λ be a local commuting Hamiltonian with $\beta < \beta_c$ and such that one of the following conditions holds:

1. H_Λ is classical.
2. H_Λ is a nearest neighbour Hamiltonian.
3. Λ is 1D.

Then, there exists a local quantum Markov semigroup with fixed point σ_Λ, the Gibbs state of H_Λ, such that it has a positive **MLSI constant** which is independent of the system size.

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Main result

MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '21)

Let $\mathcal{L}_\Lambda^{D;*}$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^{D;*}$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^{D;*}) = \Omega(\ln(|\Lambda|)^{-1})$.

Rapid mixing:

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \| \rho_t - \sigma_\Lambda \|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

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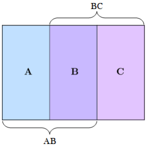
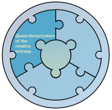
Proof: Conditional relative entropies + Quasi-factorization





Conditional relative entropies:
$$D_A(\rho_\Lambda\|\sigma_\Lambda) := D(\rho_\Lambda\|\sigma_\Lambda) - D(\rho_{A^c}\|\sigma_{A^c}) \,,$$
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Quasi-factorization (C.-Lucia-Pérez García '18)

Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

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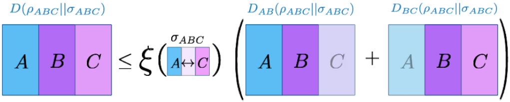

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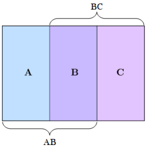

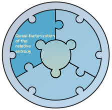

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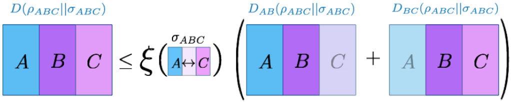

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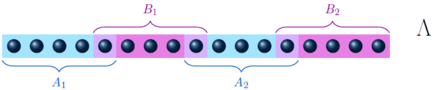

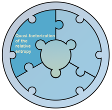

$\sigma_\Lambda = \dfrac{e^{-\beta H_\Lambda}}{\operatorname{tr}(e^{-\beta H_\Lambda})}$ is the Gibbs state of a k-local, commuting Hamiltonian H_Λ.

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Since σ_Λ is a QMC between $A_i \leftrightarrow \partial(A_i) \leftrightarrow (A_i \cup \partial A_i)^c$, then:

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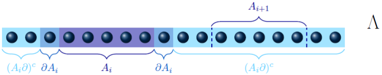

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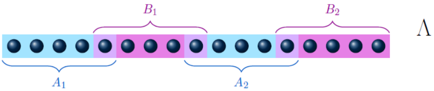

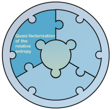

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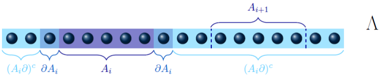
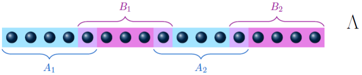
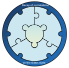


Proof: Decay of correlations





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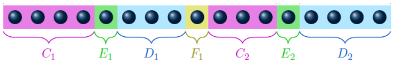

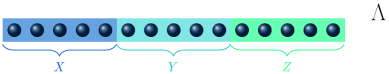

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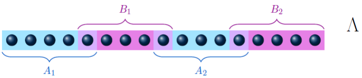

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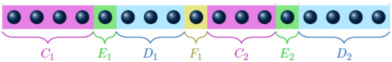

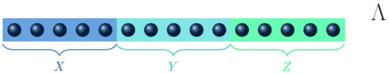

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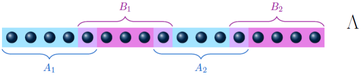

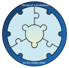

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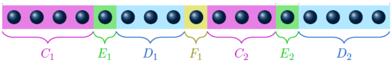

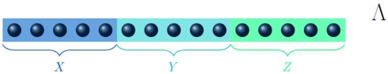

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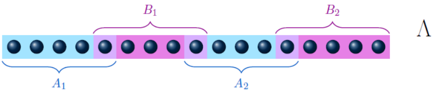
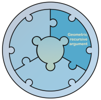
Proof: Geometric recursive argument





Let us recall: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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Comparison between conditional relative entropies (Bardet-C.-Rouzé, '20)

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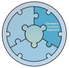
Therefore, by this and  +  , we have:

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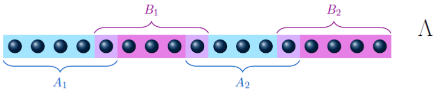
and thus

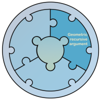
$$\alpha(\mathcal{L}_\Lambda^{H;*}) \geq \frac{K}{\xi(\sigma_{A^c B^c})} \min \left\{ \alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}), \alpha_{B_i}(\mathcal{L}_\Lambda^{H;*}) \right\} ,$$

for

$$\alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}) = \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{- \operatorname{tr}\left[\mathcal{L}_{A_i}^{H;*}(\rho_\Lambda)(\ln \rho_\Lambda - \ln \sigma_\Lambda) \right]}{D(\rho_\Lambda \| E_{A_i}^*(\rho_\Lambda))} .$$

PROOF: GEOMETRIC RECURSIVE ARGUMENT





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COMPARISON BETWEEN CONDITIONAL RELATIVE ENTROPIES (Bardet-C.-Rouzé, '20)

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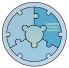
Therefore, by this and  +  , we have:

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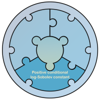
Proof: Positive CMLSI



Reduction of conditional relative entropies (Gao-Rouzé, '21)

$$D(\rho_\Lambda \| E^*_{A_i}(\rho_\Lambda)) \leq 4k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E^*_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda},$$

where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

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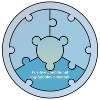

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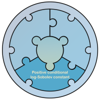

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The CMLSI of the local generators is positive:

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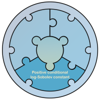

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Davies and heat-bath dynamics (Bardet-C.-Rouzé, '20)

The conditional expectations associated to Davies and heat-bath dynamics coincide.

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