Quantum Logarithmic Sobolev Inequalities for Quantum Many-Body Systems

Ángela Capel (Universität Tübingen)

UNED Applied Mathematics Seminar, 18 November 2021

Quantum

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Communication channels $\longleftrightarrow$ Physical interactions

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Dissipative evolutions of quantum many-body systems

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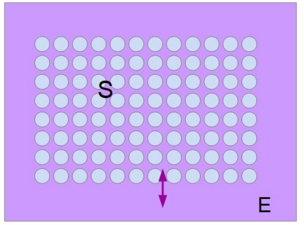

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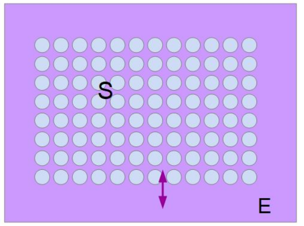

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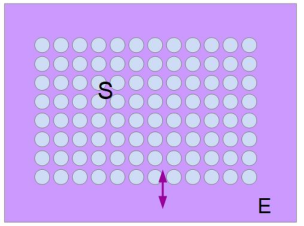

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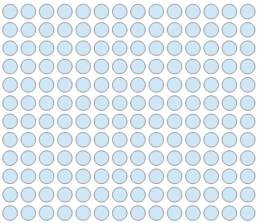

Figure: A quantum spin lattice system.

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- To every site $x \in \Lambda$ we associate $\mathcal{H}_x$ $(= \mathbb{C}^D)$.
- The global Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$.
- The set of bounded linear endomorphisms on $\mathcal{H}_\Lambda$ is denoted by $\mathcal{B}_\Lambda := \mathcal{B}(\mathcal{H}_\Lambda)$.
- The set of density matrices is denoted by $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \, : \, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

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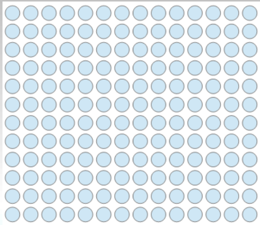

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Postulates of quantum mechanics

Postulate 1

Given an isolated physical system, there is a complex Hilbert space $\mathcal{H}$ associated to it, which is known as the **state space** of the system.

Moreover, the physical system is completely described by its **state vector**, which is a unitary vector in the state space.

Postulate 2

Given an isolated physical system, its evolution is described by a **unitary transformation** in the Hilbert space.

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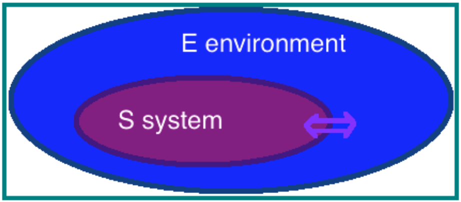

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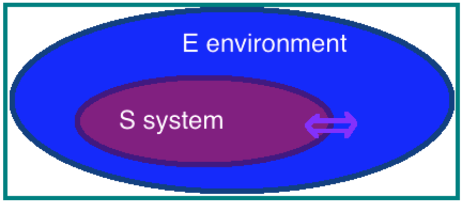

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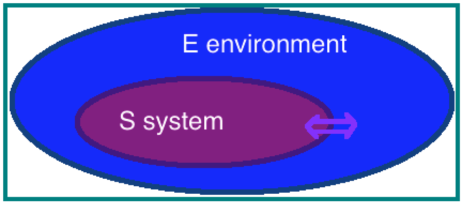

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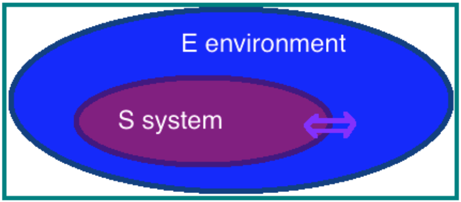

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Dissipative quantum systems

Primitive QMS

We assume that $\{\mathcal{T}_t^*\}_{t\geq 0}$ has a unique full-rank invariant state which we denote by σ_Λ.

Reversibility

We also assume that the quantum Markov process studied is **reversible**, i.e., satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}(g)\rangle_\sigma = \langle \mathcal{L}(f), g\rangle_\sigma ,$$

for every $f, g \in \mathcal{B}_\Lambda$ and Hermitian, where

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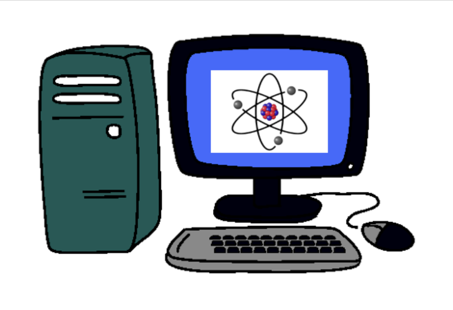

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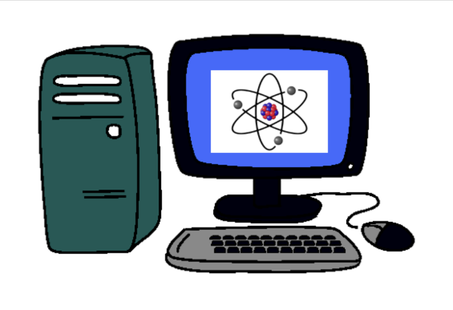

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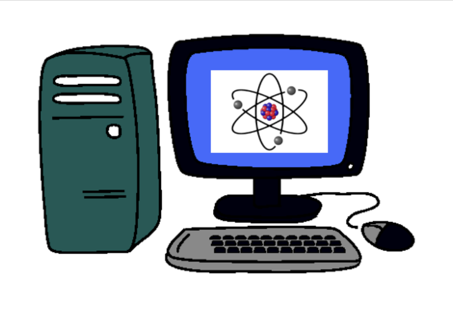

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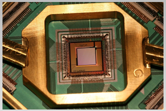

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New area:

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to create artificial evolutions in which the dissipative process works in favor (protecting the system from noisy evolutions).

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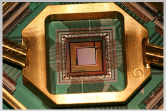

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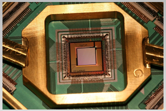

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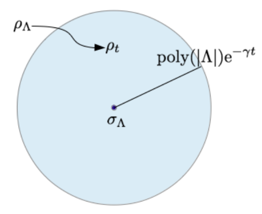

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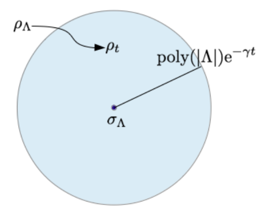

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Relative entropy: of ρ_t and σ_Λ:

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$$\partial_t \rho_t = \mathcal{L}_\Lambda^*(\rho_t).$$

Relative entropy of ρ_t and σ_Λ:

$$D(\rho_t||\sigma_\Lambda) = \mathrm{tr}[\rho_t(\log \rho_t - \log \sigma_\Lambda)].$$

Differentiating:

$$\partial_t D(\rho_t||\sigma_\Lambda) = \mathrm{tr}[\mathcal{L}_\Lambda^*(\rho_t)(\log \rho_t - \log \sigma_\Lambda)].$$

Modified log-Sobolev inequality (MLSI)

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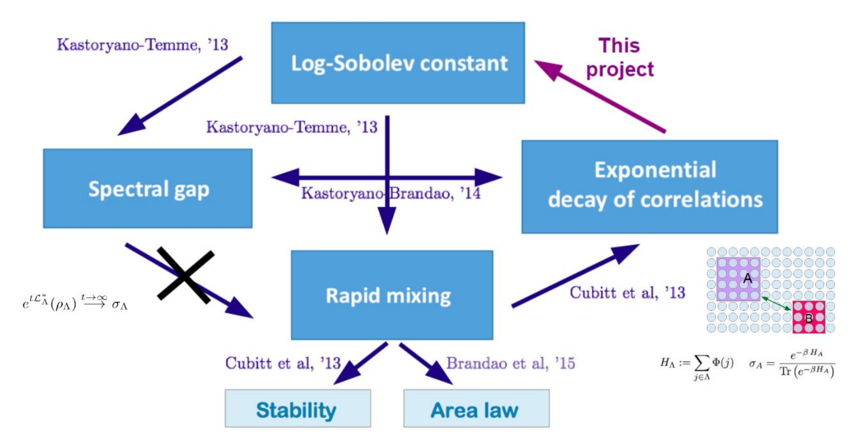
Quantum spin systems



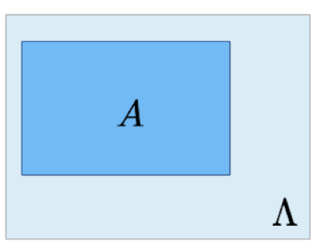
Exp. decay of correlations:

$$\sup_{\|O_A\|=\|O_B\|=1} |\mathrm{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]| \leq K\, \mathrm{e}^{-\gamma d(A,B)} .$$

Objective

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}^*_\Lambda) \geq \Psi(|\Lambda|) > 0.$$



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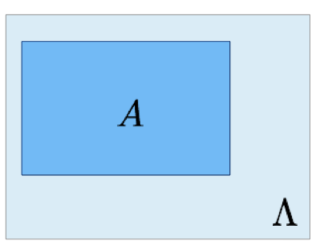

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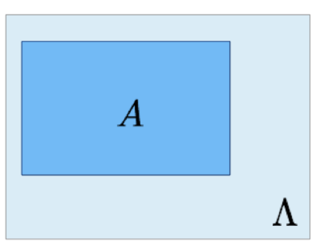

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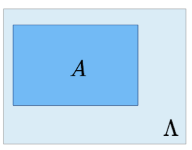
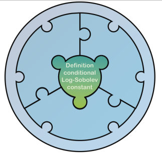
Conditional MLSI constant





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The **MLSI constant** of $\mathcal{L}_\Lambda^* = \sum_{k \in \Lambda} \mathcal{L}_k^*$ is defined by

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

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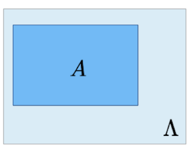

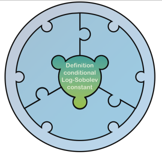

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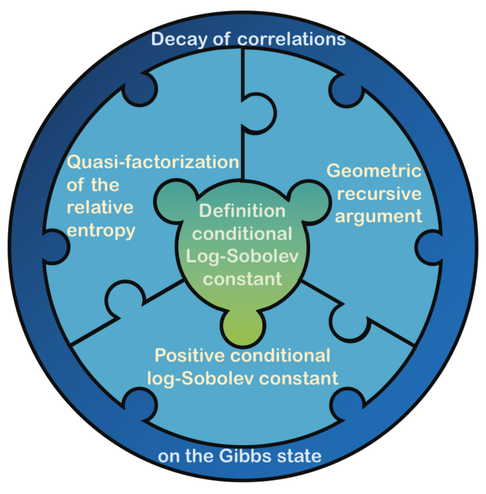
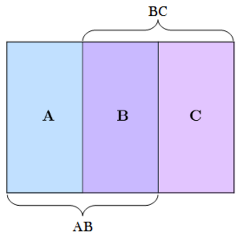
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

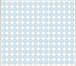
$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Example: Tensor product fixed point

(C.-Lucia-Pérez García '18) $\mathcal{L}^*_\Lambda(\rho_\Lambda) = \sum_{x\in\Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$ **heat-bath**

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

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$\alpha_\Lambda(\mathcal{L}^*_x) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2 D_x(\rho_\Lambda||\sigma_\Lambda)}$

$$\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}^*_x)}$$

$$\leq \frac{1}{2 \inf_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}^*_x)} \sum_{x\in\Lambda} -\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]$$

$$= \frac{1}{2 \inf_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}^*_x)} \left(-\operatorname{tr}[\mathcal{L}^*_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right)$$

$$\leq \left(-\operatorname{tr}[\mathcal{L}^*_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]\right).$$

Dynamics

Let $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}\left[e^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

$$\mathcal{L}_\Lambda^{H;*}(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right)$$

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The **Davies generator** is given by:

$$\mathcal{L}_\Lambda^{D}(X) := i[H_\Lambda, X] + \sum_{x \in \Lambda} \mathcal{L}_x^D(X),$$

where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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Let $\mathcal{L}_\Lambda^{H,D;*}$ be the **heat-bath** or **Davies** generator in 1D. Then, $\mathcal{L}_\Lambda^{H,D;*}$ has a positive spectral gap that is independent of the system size, for every temperature.

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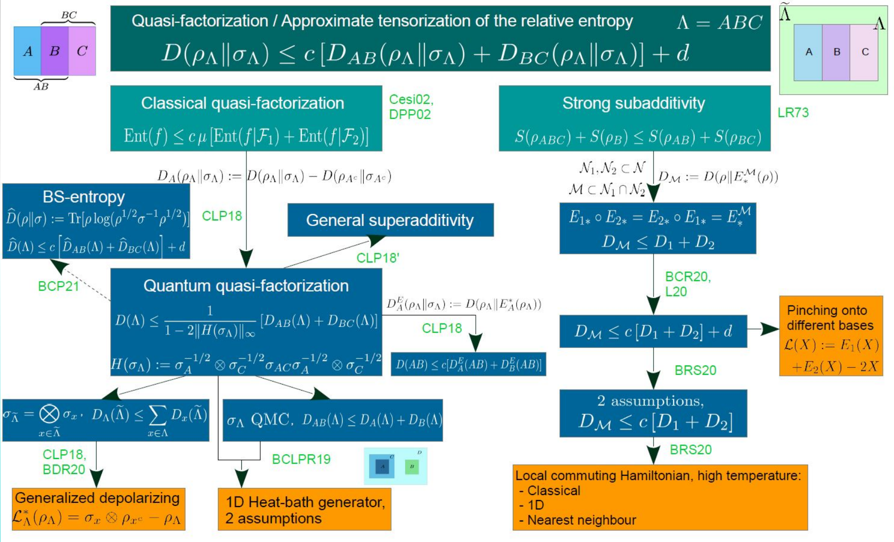

Most recent result

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For $\alpha(\mathcal{L}_\Lambda^*)$ a **MLSI constant**:

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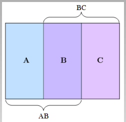

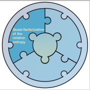

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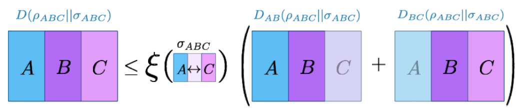

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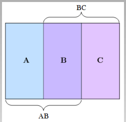

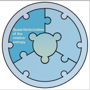

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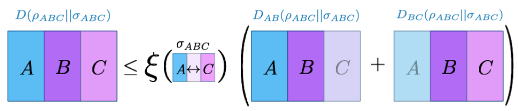

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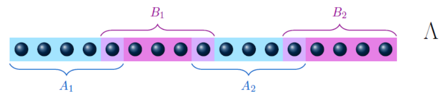

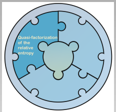

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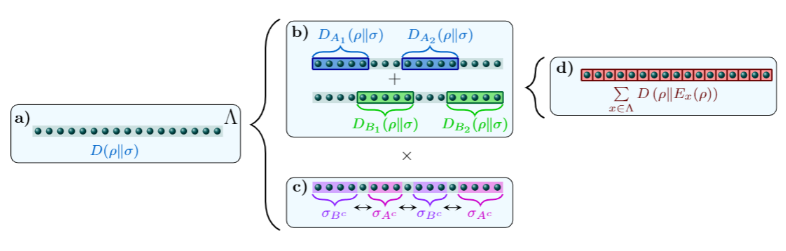

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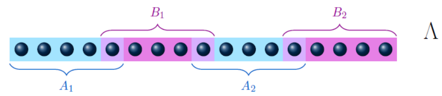

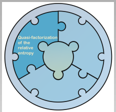

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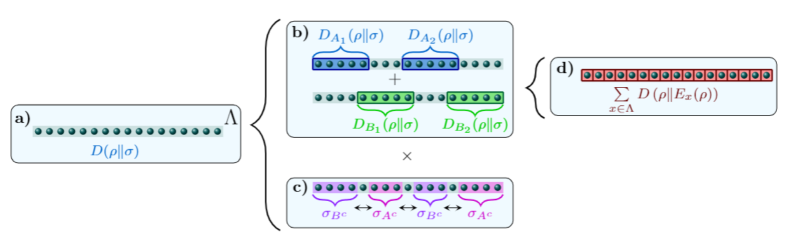

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Do you have any questions?

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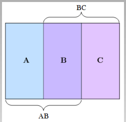

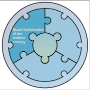

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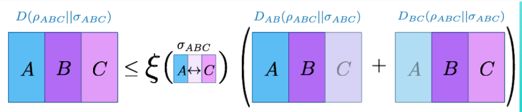

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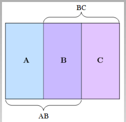

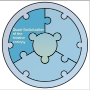

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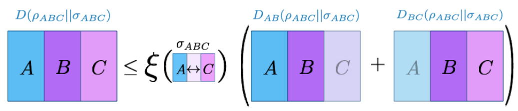

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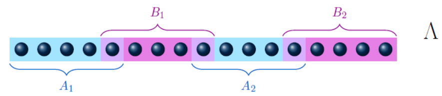

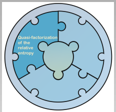

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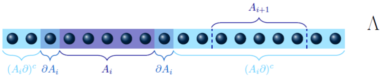

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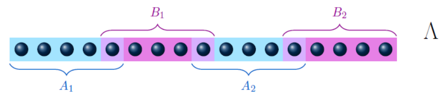

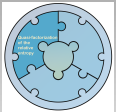

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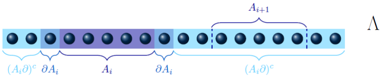

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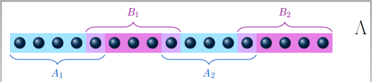

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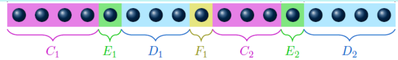

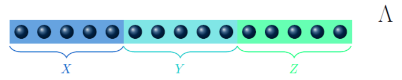

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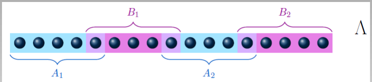

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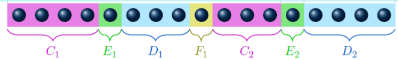

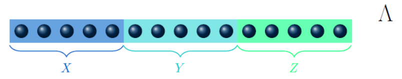

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As a consequence, $\xi(\sigma_{A^c B^c})$ is uniformly bounded as long as # segments $= \mathcal{O}(|\Lambda| / \ln |\Lambda|)$.

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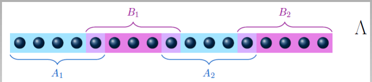


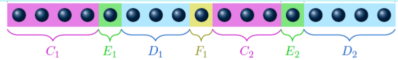
Quasi-factorization

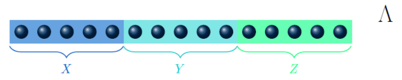
Let $A \cup B = \Lambda \subset \mathbb{Z}$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$. The following holds

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \xi(\sigma_{A^c B^c}) \sum_i \left[D_{A_i}(\rho_\Lambda || \sigma_\Lambda) + D_{B_i}(\rho_\Lambda || \sigma_\Lambda) \right],$$

where

$$\xi(\sigma_{A^c B^c}) = \frac{1}{1 - 2 \left\| \sigma_{A^c}^{-1/2} \otimes \sigma_{B^c}^{-1/2} \, \sigma_{A^c B^c} \, \sigma_{A^c}^{-1/2} \otimes \sigma_{B^c}^{-1/2} - \mathbb{1}_{A^c B^c} \right\|_\infty}.$$





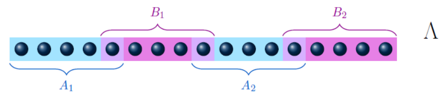
Decay of correlations, (Bluhm-C.-Pérez Hernández, '21)

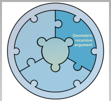
Let σ_{XYZ} be the Gibbs state of a finite-range, translation-invariant Hamiltonian. There is $\ell \mapsto \delta(\ell)$ with exponential decay such that:

$$\left\| \sigma_X^{-1} \otimes \sigma_Z^{-1} \sigma_{XZ} - \mathbb{1}_{XZ} \right\|_\infty \leq \delta(|Y|).$$

As a consequence, $\xi(\sigma_{A^c B^c})$ is uniformly bounded as long as # segments $= \mathcal{O}(|\Lambda| / \ln |\Lambda|)$.

Proof: Geometric recursive argument

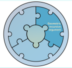




Let us recall: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
$D_A^E(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| E_A^*(\rho_\Lambda))$.

Comparison between conditional relative entropies (Bardet-C.-Rouzé, '20)

$$D_A(\rho_\Lambda \| \sigma_\Lambda) \leq D_A^E(\rho_\Lambda \| \sigma_\Lambda)$$

Therefore, by this and  +  , we have:

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and thus

$$\alpha(\mathcal{L}_\Lambda^{H;*}) \geq \frac{K}{\xi(\sigma_{A^c B^c})} \min \left\{ \alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}), \alpha_{B_i}(\mathcal{L}_\Lambda^{H;*}) \right\},$$

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$$\alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}) = \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{- \operatorname{tr}\left[\mathcal{L}_{A_i}^{H;*}(\rho_\Lambda)(\ln \rho_\Lambda - \ln \sigma_\Lambda) \right]}{D(\rho_\Lambda \| E_{A_i}^*(\rho_\Lambda))} .$$

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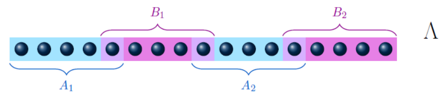

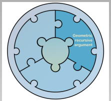

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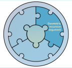
Therefore, by this and  +  , we have:

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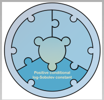
Proof: Positive CMLSI



Reduction of conditional relative entropies (Gao-Rouzé, '21)

$$D(\rho_\Lambda \| E^*_{A_i}(\rho_\Lambda)) \leq 4k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E^*_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda},$$

where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

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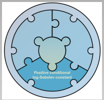

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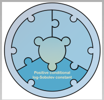

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CMLSI (Gao-Rouzé, '21)

The CMLSI of the local generators is positive:

$$\alpha_c(\mathcal{L}_j^{D;*}) := \inf_{k \in \mathbb{N}} \alpha(\mathcal{L}_j^{D;*} \otimes \mathrm{Id}_k) > 0 .$$

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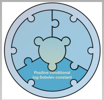

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Last step

Heat-bath cond. expectation: $E_A^{H;*}(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2}\sigma_{A^c}^{-1/2}\,\mathrm{tr}_A[\,\cdot\,]\,\sigma_{A^c}^{-1/2}\sigma_\Lambda^{1/2}\right)^n .$

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The conditional expectations associated to Davies and heat-bath dynamics coincide.

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Conclusion

For $\mathcal{L}_\Lambda^{D;*}$, there is a positive MLSI constant $\alpha(\mathcal{L}_\Lambda^{D;*}) = \Omega(\ln|\Lambda|^{-1})$.
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