Approximate tensorization for the relative entropy and exponential decay for the mutual information

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Technical University of Munich

Munich Center for Quantum Science and Technology

Open quantum systems

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Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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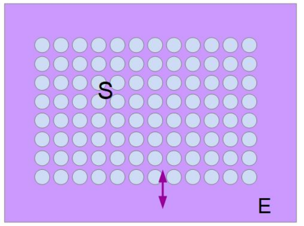

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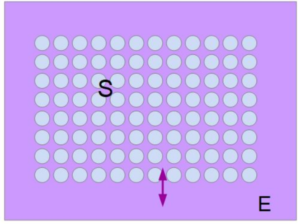

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- The continuous-time evolution of a state on S is given by a q. Markov semigroup (**Markovian approximation**).

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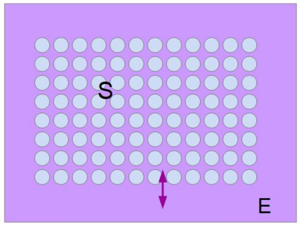

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Quantum Markov semigroups

Quantum Markov semigroups

A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

$$\rho_\Lambda \xrightarrow{t} \rho_t := \mathcal{T}_t^*(\rho_\Lambda) = e^{t\mathcal{L}_\Lambda^*}(\rho_\Lambda) \xrightarrow{t\to\infty} \sigma_\Lambda$$

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Rapid mixing

We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

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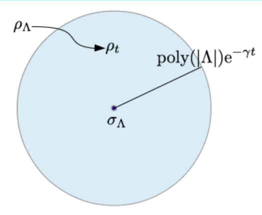

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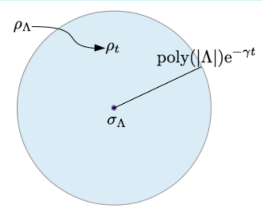

Modified logarithmic Sobolev inequality

MLSI constant

The **MLSI constant** of $\mathcal{L}_\Lambda^*$ is defined as:

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) > 0$:

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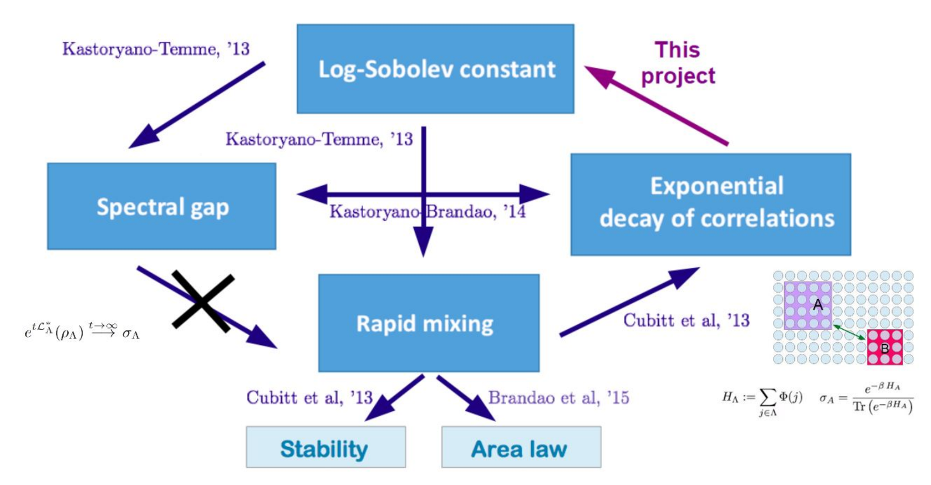
Quantum spin systems



Classical spin systems

(Cesi, Dai Pra-Paganoni-Posta, '02)

(1) Quasi-factorization of the entropy (in terms of a conditional entropy).

+

(2) Recursive geometric argument.
Lower bound for the global log-Sobolev constant in terms of the log-Sobolev constant of a size-fixed region.

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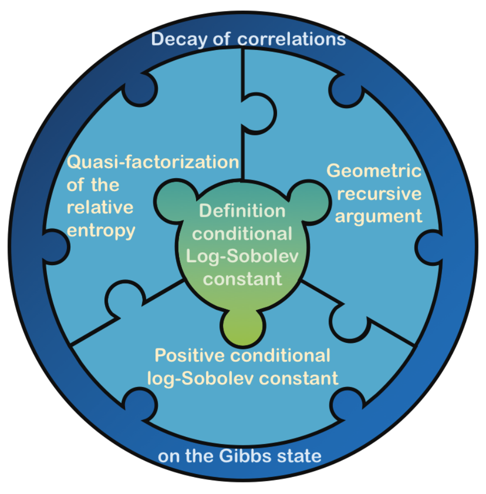
$\Downarrow$

Positive log-Sobolev constant.

Strategy

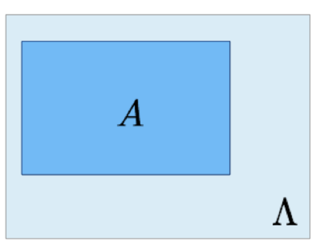
Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Objective

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda^*) \geq \Psi(|\Lambda|) > 0.$$



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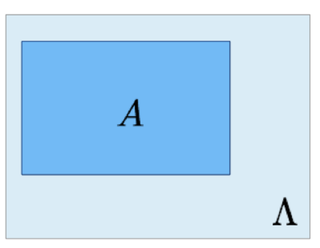

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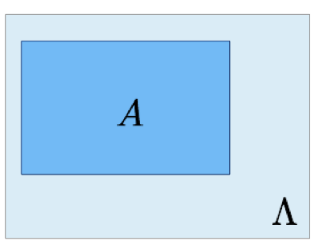

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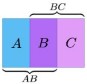
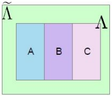
Quasi-factorization / Approximate tensorization



Quasi-factorization / Approximate tensorization of the relative entropy $\Lambda = ABC$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq c \left[D_{AB}(\rho_\Lambda || \sigma_\Lambda) + D_{BC}(\rho_\Lambda || \sigma_\Lambda) \right] + d$$



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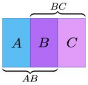

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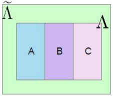

Classical quasi-factorization

$$\mathrm{Ent}(f) \leq c\, \mu\left[\mathrm{Ent}(f|\mathcal{F}_1) + \mathrm{Ent}(f|\mathcal{F}_2)\right]$$

Cesi02, DPP02

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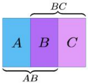

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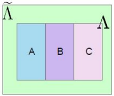

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Strong subadditivity

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC})$$

LR73

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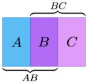

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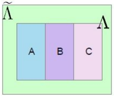

Classical quasi-factorization (Cesi02, DPP02)

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$$D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$$

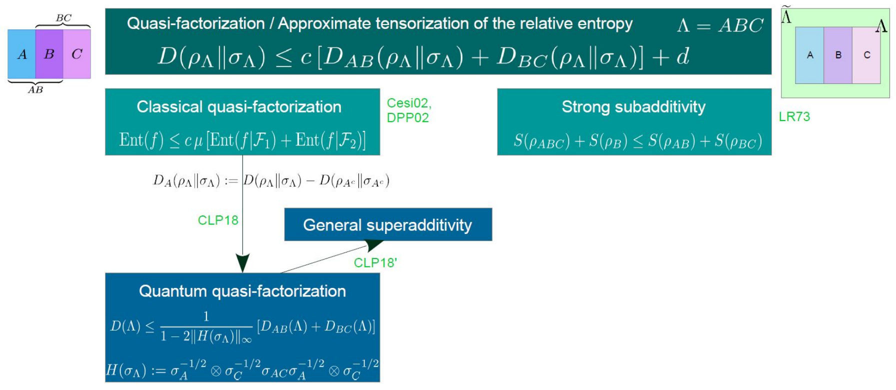
CLP18

Quantum quasi-factorization

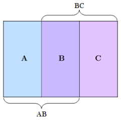
$$D(\Lambda) \leq \frac{1}{1 - 2\|H(\sigma_\Lambda)\|_\infty} [D_{AB}(\Lambda) + D_{BC}(\Lambda)]$$

$$H(\sigma_\Lambda) := \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \sigma_{AC} \sigma_A^{-1/2} \otimes \sigma_C^{-1/2}$$

Quasi-factorization / Approximate tensorization



Quasi-factorization for the relative entropy



$$D_A(\rho_{ABC}||\sigma_{ABC}) := D(\rho_{ABC}||\sigma_{ABC}) - D(\rho_{BC}||\sigma_{BC})$$

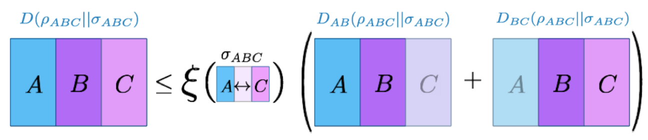
Quasi-factorization for the CRE (C.-Lucia-Pérez García '18)

Let $\mathcal{H}_{ABC}$ and $\rho_{ABC}, \sigma_{ABC} \in \mathcal{S}_{ABC}$. The following holds

$$D(\rho_{ABC}||\sigma_{ABC}) \leq \xi(\sigma_{AC}) \left[D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC}) \right],$$

where

$$\xi(\sigma_{AC}) = \frac{1}{1 - 2 \left\| \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \, \sigma_{AC} \, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC} \right\|_\infty}.$$



General superadditivity for the relative entropy

$$(1-2\|H(\sigma_{AC})\|_\infty) D(\rho_{ABC}||\sigma_{ABC}) \leq D_{AB}(\rho_{ABC}||\sigma_{ABC}) + D_{BC}(\rho_{ABC}||\sigma_{ABC})$$
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The previous result is equivalent to (C.-Lucia-Pérez García '18):

$$\boxed{(1+2\|H(\sigma_{AB})\|_\infty)D(\rho_{AB}||\sigma_{AB}) \geq D(\rho_A||\sigma_A) + D(\rho_B||\sigma_B)}.$$

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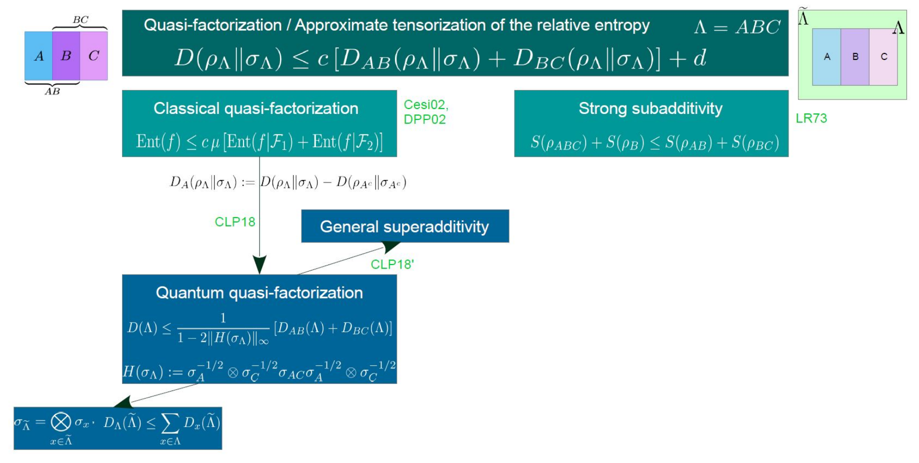

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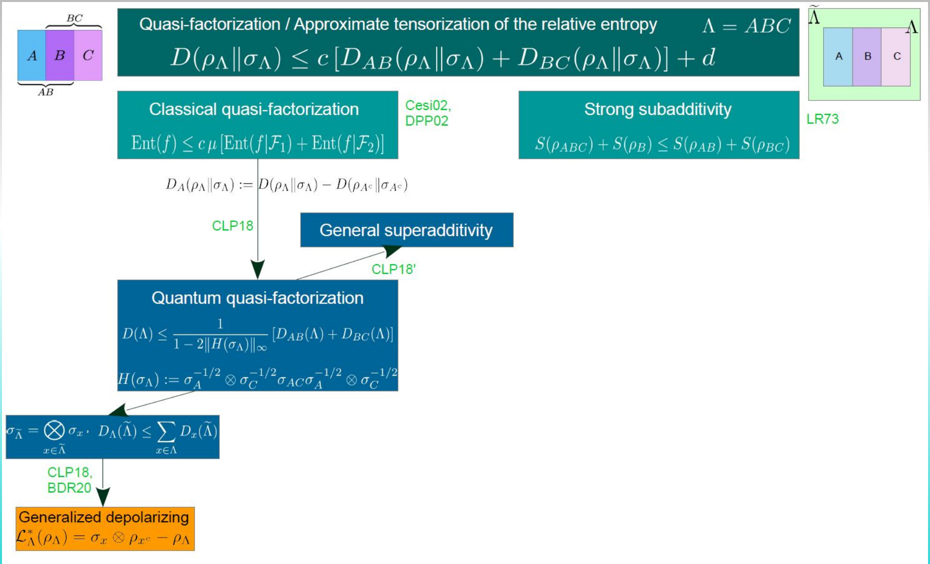

Heat-bath with tensor product fixed point

Consider the local and global Lindbladians

$$\mathcal{L}_x^* := \mathbb{E}_x^* - \mathbb{1}_\Lambda , \quad \mathcal{L}_\Lambda^* = \sum_{x \in \Lambda} \mathcal{L}_x^*$$

Since

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The **heat-bath dynamics**, with tensor product fixed point, has MLSI(1/2).

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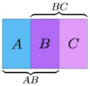
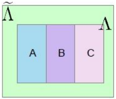
Quasi-factorization / Approximate tensorization



Quasi-factorization / Approximate tensorization of the relative entropy $\Lambda = ABC$

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq c \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] + d$$



Classical quasi-factorization (Cesi02, DPP02)

$$\mathrm{Ent}(f) \leq c\, \mu \left[\mathrm{Ent}(f|\mathcal{F}_1) + \mathrm{Ent}(f|\mathcal{F}_2) \right]$$

Strong subadditivity (LR73)

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC})$$

$$D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$$

CLP18

General superadditivity (CLP18')

Quantum quasi-factorization

$$D(\Lambda) \leq \frac{1}{1 - 2\| H(\sigma_\Lambda) \|_\infty} \left[D_{AB}(\Lambda) + D_{BC}(\Lambda) \right]$$

$$H(\sigma_\Lambda) := \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \sigma_{AC} \sigma_A^{-1/2} \otimes \sigma_C^{-1/2}$$

$$\sigma_{\widetilde{\Lambda}} = \bigotimes_{x \in \widetilde{\Lambda}} \sigma_x, \quad D_\Lambda(\widetilde{\Lambda}) \leq \sum_{x \in \Lambda} D_x(\widetilde{\Lambda})$$

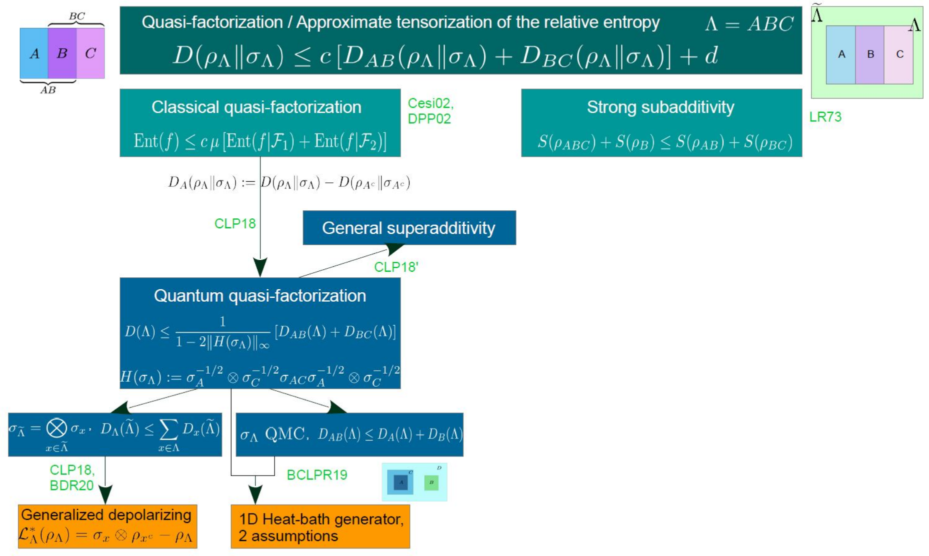
$$\sigma_\Lambda \text{ QMC}, \quad D_{AB}(\Lambda) \leq D_A(\Lambda) + D_B(\Lambda)$$

CLP18, BDR20

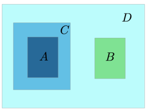
Generalized depolarizing

$$\mathcal{L}_\Lambda^*(\rho_\Lambda) = \sigma_x \otimes \rho_{x^c} - \rho_\Lambda$$

Quasi-factorization / Approximate tensorization



Heat-bath dynamics in 1D



$\sigma_\Lambda = \frac{e^{-\beta H}}{\text{tr}(e^{-\beta H})}$ is the Gibbs state of a k-local, commuting Hamiltonian H.

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$$\mathcal{L}^*_\Lambda(\rho_\Lambda) = \sum_{x \in \Lambda} \mathcal{L}^*_x(\rho_\Lambda) = \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right) .$$

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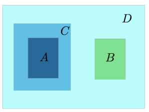

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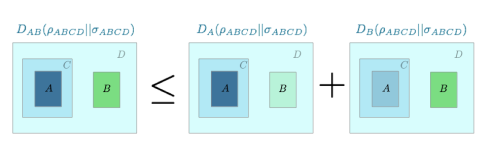
Quasi-factorization for quantum Markov chains (Bardet-C.-Lucia-Pérez García-Rouzé'19)

Let $\mathcal{H}_{ABCD} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \otimes \mathcal{H}_D$, where C shields A from B and D, and let $\rho_{ABCD}, \sigma_{ABCD} \in \mathcal{S}_{ABCD}$. Assume that σ_{ABCD} is a QMC between $A \leftrightarrow C \leftrightarrow BD$. Then, the following inequality holds:

$$D_{AB}(\rho_{ABCD}||\sigma_{ABCD}) \leq D_A(\rho_{ABCD}||\sigma_{ABCD}) + D_B(\rho_{ABCD}||\sigma_{ABCD}).$$

$$\sigma_\Lambda = \bigoplus_{i\in I} \sigma_{A(\partial c)_i^L} \otimes \sigma_{(\partial c)_i^R BD}$$



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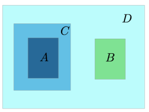

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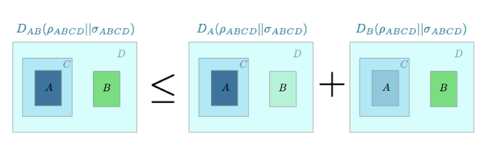

Heat-bath dynamics in 1D

Assumption 1

In a tripartite Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_C \otimes \mathcal{H}_B$, A and B not connected, we have

$$\left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty \leq K < \frac{1}{2}.$$

In particular, Gibbs states at high enough temperature satisfy this.

Assumption 2

For any $B \subset \Lambda$, $B = B_1 \cup B_2$, it holds:

$$D_B(\rho_\Lambda || \sigma_\Lambda) \leq f(\sigma_{B\partial}) \left(D_{B_1}(\rho_\Lambda || \sigma_\Lambda) + D_{B_2}(\rho_\Lambda || \sigma_\Lambda) \right).$$

In particular, tensor products satisfy this (with $f = 1$).

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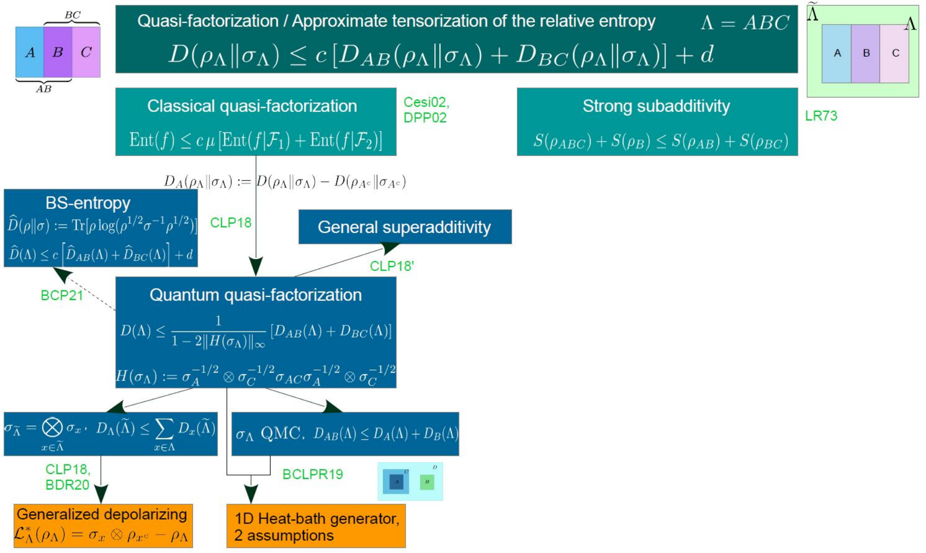

BS-entropy

$$\widehat{D}(\rho_{AB}||\sigma_{AB}) := \operatorname{tr}\left[\rho_{AB} \log\left(\rho_{AB}^{1/2}\,\sigma_{AB}^{-1}\,\rho_{AB}^{1/2}\right)\right], \quad \widehat{D}_A(\rho_{AB}||\sigma_{AB}) := \widehat{D}(\rho_{AB}||\sigma_{AB}) - \widehat{D}(\rho_B||\sigma_B).$$

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Theorem (Bluhm-C.-Pérez Hernández '21)

Let $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ and $\rho_{AB}, \sigma_{AB} \in \mathcal{S}_{AB}$. The following inequality holds whenever $\|H(\sigma_{AB})\|_\infty < 1/2$:

$$\widehat{D}(\rho_{AB}||\sigma_{AB}) \leq \widetilde{M}(\sigma_{AB})\left[\widehat{D}_A(\rho_{AB}||\sigma_{AB}) + \widehat{D}_B(\rho_{AB}||\sigma_{AB})\right] + \widetilde{L}(\rho_{AB}, \sigma_{AB}),$$

where

$$\widetilde{M}(\sigma_{AB}) := \frac{1}{1 - 2\|H(\sigma_{AB})\|_\infty},$$

and

$$\widetilde{L}(\rho_{AB}, \sigma_{AB}) \leq f\left(\left\|\left[\rho_A^{1/2}, \sigma_A^{-1/2}\right]\right\|_\infty, \left\|\left[\rho_B^{1/2}, \sigma_B^{-1/2}\right]\right\|_\infty\right).$$

Note that if $\sigma_{AB} = \sigma_A \otimes \sigma_B$, we have $\widetilde{M}(\sigma_{AB}) = 1$, and if $\rho_A^{1/2}\sigma_A^{-1/2}$ and $\rho_B^{1/2}\sigma_B^{-1/2}$ are normal (in particular, if $[\rho_A, \sigma_A] = [\rho_B, \sigma_B] = 0$), then $\widetilde{L}(\rho_{AB}, \sigma_{AB}) = 0$.

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$$\widetilde{L}(\rho_{AB},\sigma_{AB}) \leq f\left(\left\|\left[\rho_A^{1/2},\sigma_A^{-1/2}\right]\right\|_\infty, \left\|\left[\rho_B^{1/2},\sigma_B^{-1/2}\right]\right\|_\infty\right).$$

Note that if $\sigma_{AB} = \sigma_A \otimes \sigma_B$, we have $\widetilde{M}(\sigma_{AB}) = 1$, and if $\rho_A^{1/2}\sigma_A^{-1/2}$ and $\rho_B^{1/2}\sigma_B^{-1/2}$ are normal (in particular, if $[\rho_A,\sigma_A] = [\rho_B,\sigma_B] = 0$), then $\widetilde{L}(\rho_{AB},\sigma_{AB}) = 0$.

BS-entropy

If $\widetilde{L}(\rho_{AB}, \sigma_{AB}) = 0$ in general, the previous result would be equivalent to superadditivity for the BS-entropy.

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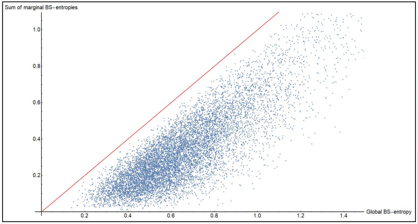

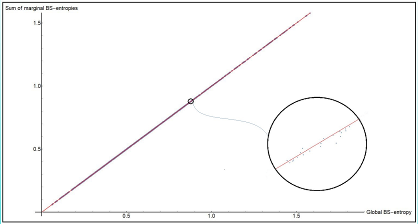

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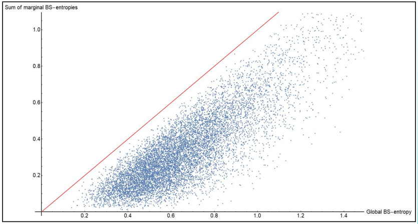
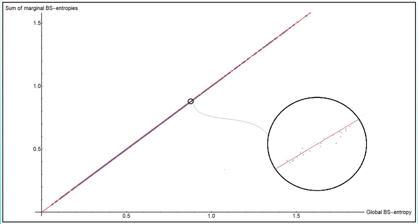
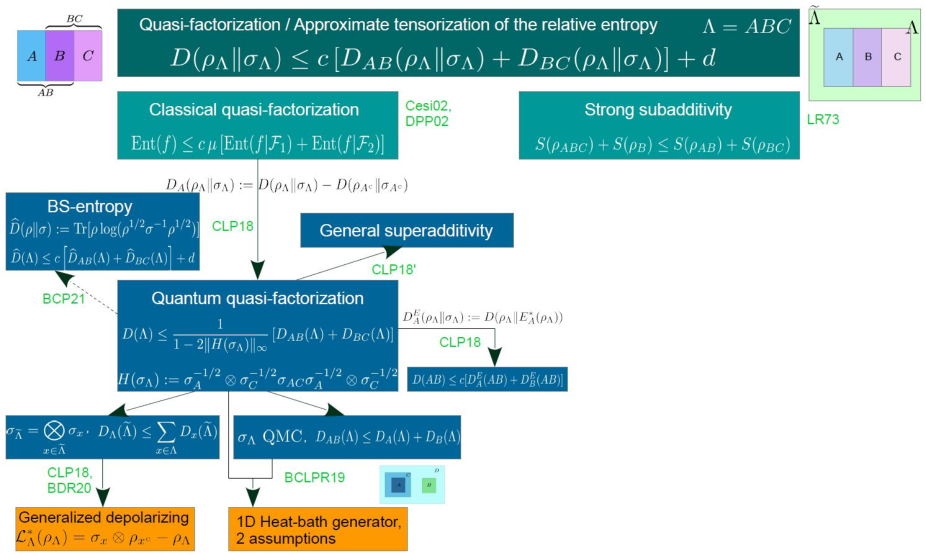




Quasi-factorization / Approximate tensorization



$$D_A^E(\rho_\Lambda\|\sigma_\Lambda) := D(\rho_\Lambda\|\sigma_\Lambda^{1/2}\sigma_{A^c}^{-1/2}\rho_{A^c}\sigma_{A^c}^{-1/2}\sigma_\Lambda^{1/2}) .$$

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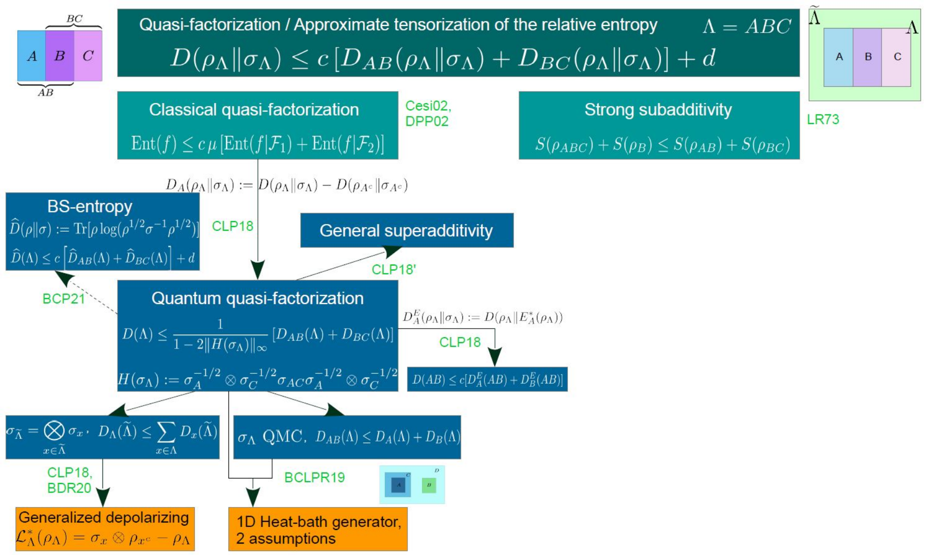

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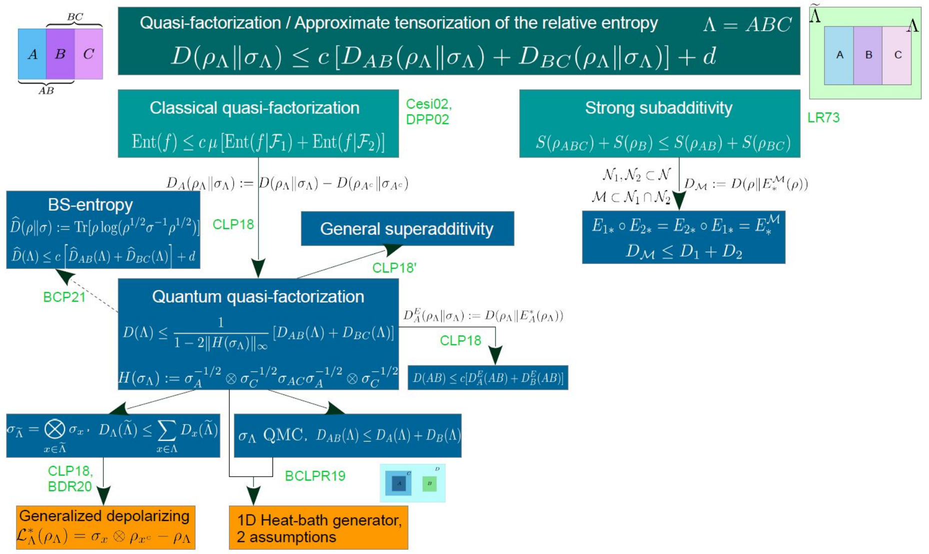

Generalization of strong subadditivity

In terms of the relative entropy, the **strong subadditivity of entropy** (Lieb-Ruskai '73) takes the form

$$D\left(\rho_{ABC} \middle\| \rho_B \otimes \frac{\mathbb{1}_{AC}}{d_{\mathcal{H}_{AC}}}\right) \leq D\left(\rho_{ABC} \middle\| \rho_{AB} \otimes \frac{\mathbb{1}_{C}}{d_{\mathcal{H}_{C}}}\right) + D\left(\rho_{ABC} \middle\| \rho_{BC} \otimes \frac{\mathbb{1}_{A}}{d_{\mathcal{H}_{A}}}\right).$$

For $\mathcal{M} \subset \mathcal{N}_1, \mathcal{N}_2 \subset \mathcal{N}$, if $E^{\mathcal{M}}, E_1, E_2$ are the conditional expectations onto $\mathcal{M}, \mathcal{N}_1, \mathcal{N}_2$, respectively, we have

$$D(\rho \| E_*^{\mathcal{M}}(\rho)) \leq D(\rho \| E_{1*}(\rho)) + D(\rho \| E_{2*}(\rho)) \Leftrightarrow E_{1*} \circ E_{2*} = E_{2*} \circ E_{1*} = E_*^{\mathcal{M}}.$$

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In general, we present conditions in (Bardet-C.-Rouzé '20) for which

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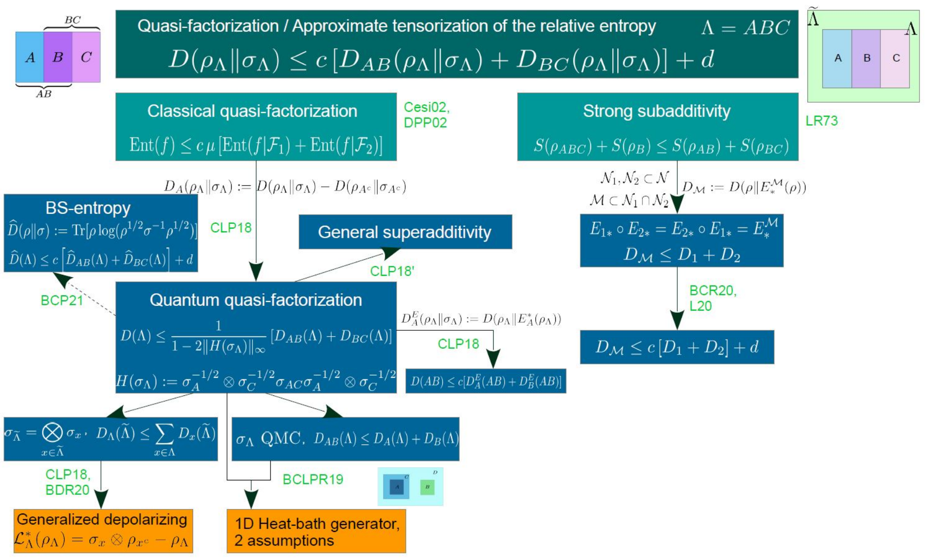

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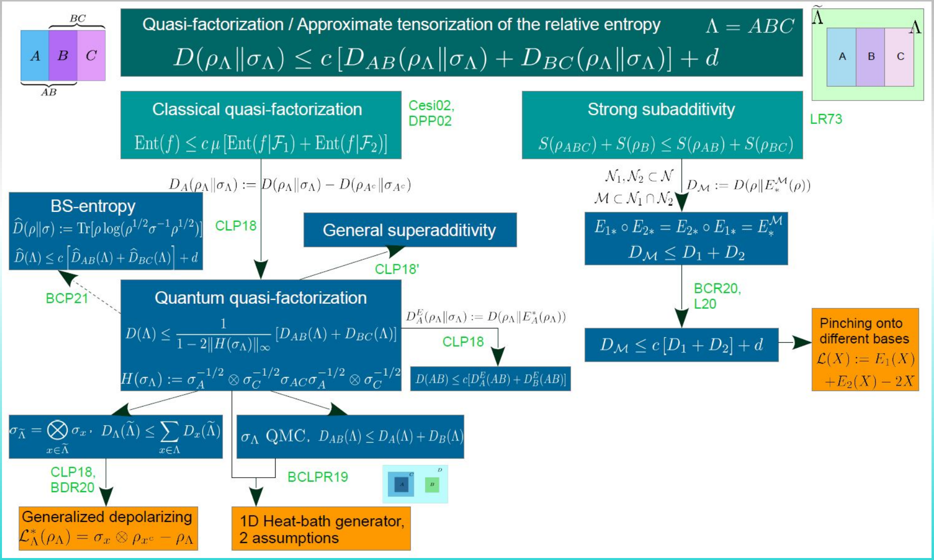

MLSI for Pinching onto different bases

$$\left\{ \left| e_k^{(1)} \right\rangle \right\} , \quad \left\{ \left| e_k^{(2)} \right\rangle \right\} \text{ orthonormal bases.}$$

$\mathcal{N}_1, \mathcal{N}_2$ diagonal onto first and second basis, respectively. $\mathcal{M} = \mathbb{C}\mathbb{1}_\ell$.

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For $i \in \{1, 2\}$, E_i denotes the Pinching map onto span $\left\{ \left| e_k^{(i)} \right\rangle \left\langle e_k^{(i)} \right| \right\}$ and $E^{\mathcal{M}} = \frac{\mathbb{1}}{\ell} \text{Tr}[\cdot]$.

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and subsequently

$$\mathcal{L}(X) := E_1(X) + E_2(X) - 2X .$$

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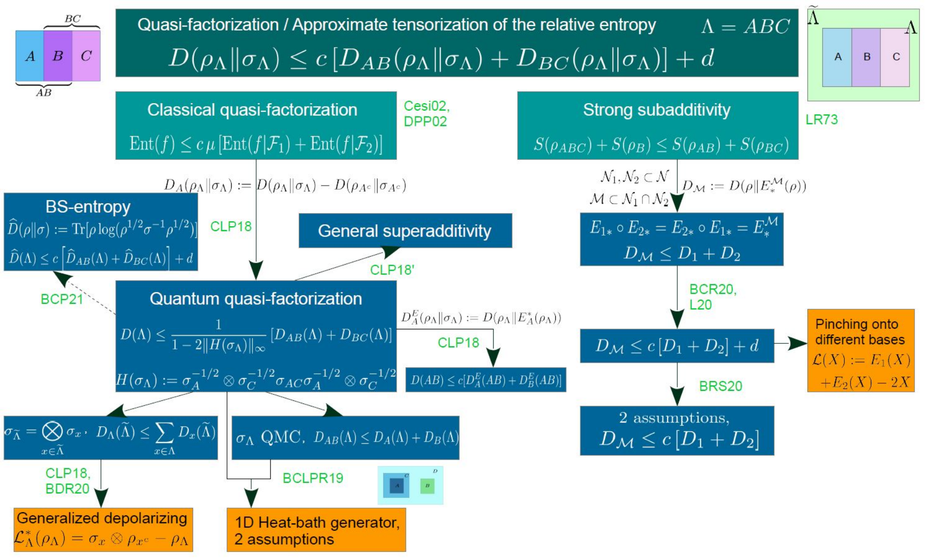

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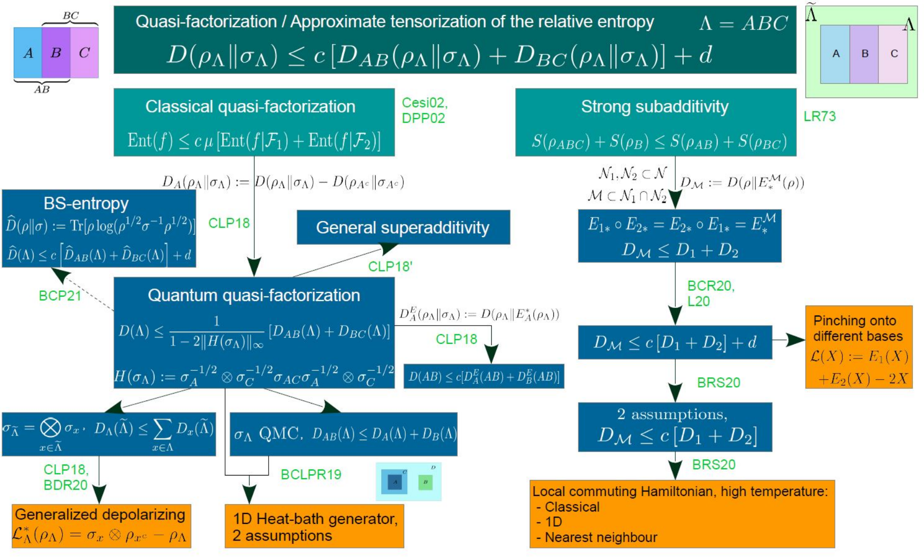
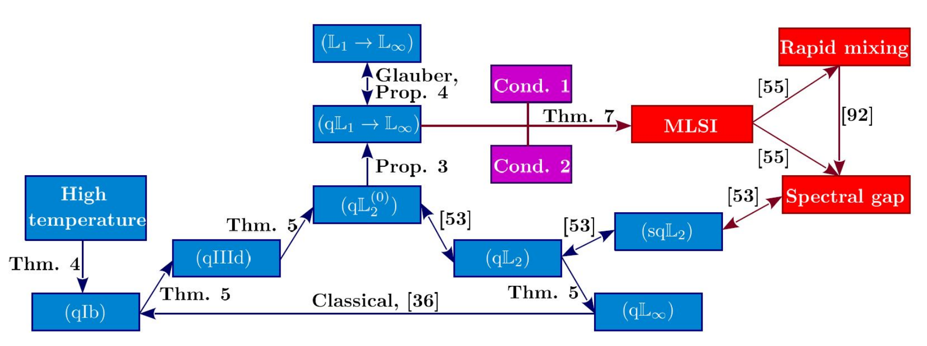


Quantum spin systems



MLSI for quantum spin systems

MLSI, informal (C.-Rouzé-Stilck França '20)

Let H_Λ be a local commuting Hamiltonian such that one of the following conditions holds:

1. H_Λ is classical for $\beta < \beta_c$.
2. H_Λ is a nearest neighbour Hamiltonian for $\beta < \beta_c$.
3. Λ is 1D and $\beta < \beta_c$.

Then, there exists a local quantum Markov semigroup with fixed point σ_Λ, the Gibbs state of H_Λ, such that it has a positive **MLSI constant** which is independent of the system size.

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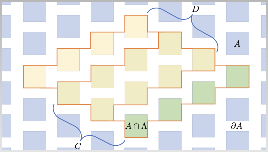
Approximate tensorization of the relative entropy



Approximate tensorization (C.-Rouzé-Stilck França '20)

Let $\mathcal{L}$ be a Gibbs sampler corresponding to a commuting potential. Assume further that the family $\mathcal{L}$ satisfies $\mathrm{q}\mathbb{L}_1 \to \mathbb{L}_\infty$ with parameters $c \geq 0$ and $\xi > 0$, as well as Condition 2. Then, for any $C, D \in \widetilde{\mathcal{S}}$ such that $C, D \subset \Lambda \subset\subset \mathbb{Z}^d$ with $2c\,|C \cup D| \exp\big(- \frac{\mathrm{d}(C\setminus D, D\setminus C)}{\xi}\big) < 1$, and all $\rho \in \mathcal{D}(\mathcal{H}_\Lambda)$,

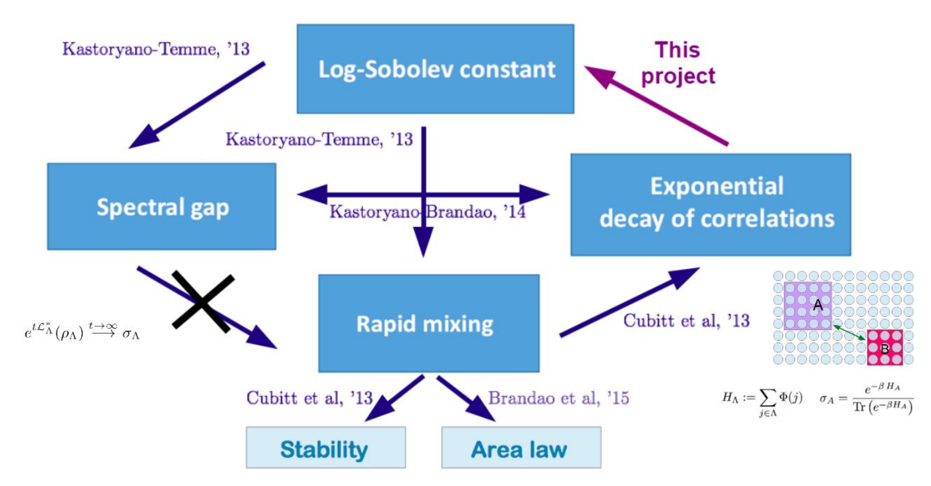
$$D(\omega \| E_{C\cup D*}(\omega)) \leq \frac{1}{1 - 2c\,|C\cup D|\,\mathrm{e}^{-\frac{\mathrm{d}(C\setminus D, D\setminus C)}{\xi}}} \Big(D(\omega\|E_{C*}(\omega)) + D(\omega\|E_{D*}(\omega)) \Big),$$

with $\omega := E_{A\cap\Lambda *}(\rho)$.

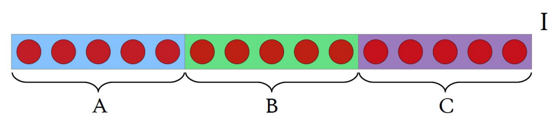
Here, we show that a condition on the **fixed points** of the generator and a condition of **decay of correlations** imply

$$d = 0,\ c \sim 1 + \kappa\,\mathrm{e}^{-\,\mathrm{d}(C\setminus D, D\setminus C)}\ .$$

Exponential decay of the mutual information in 1D



Exponential decay of correlations



Operator correlation

For a quantum state $\rho_{ABC} \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C)$, the *operator correlation* function is defined by:

$$\operatorname{Corr}_\rho(A:C) := \sup_{O_A, O_C} \left| \operatorname{Tr}[O_A \otimes O_C \, (\rho_{AC} - \rho_A \otimes \rho_C)] \right| ,$$

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Araki, '69: Any infinite 1D quantum spin system with finite range and translation invariant interactions satisfy exponential decay of correlations.

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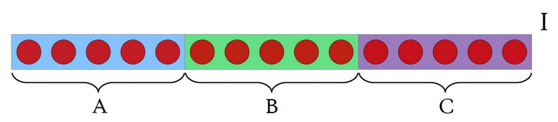

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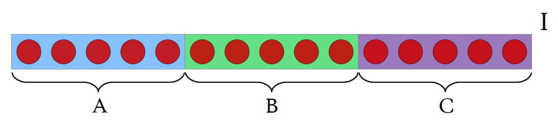

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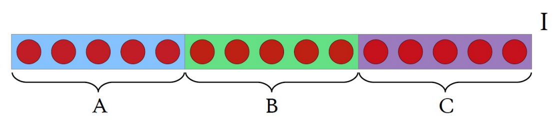

Analyticity after measurement

Given a lattice Λ and a local Hamiltonian $H = \sum_{X \subset \Lambda} \Phi_X$, its free energy is said to be δ-*analytic* for all $\beta \in [0, \beta_c)$ if it is analytic in the open ball of radius δ round β and if there exists a constant c such that for any $N \geq 0$ with $\|N\| = 1$, the following holds

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for all z_X such that $|z_X - \beta| \leq \delta$.

Harrow-Mehraban-Soleimanifar, '20: In any 1D quantum spin system, analyticity after measurement implies exponential decay of correlations on any finite interval $I = ABC$.

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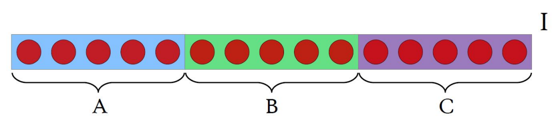

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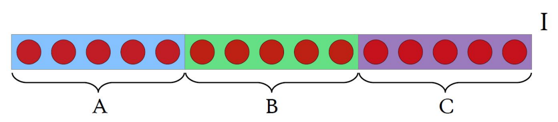

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Harrow-Mehraban-Soleimanifar, '20: In any 1D quantum spin system, analyticity after measurement implies exponential decay of correlations on any finite interval $I = ABC$.

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Given a local, finite range, non-commuting Hamiltonian in $I = ABC$ and ρ its Gibbs state, there is a positive function $\ell \mapsto \delta_1(\ell)$, depending on the local interactions and $\varepsilon(\ell)$, that exhibits exponential decay and satisfying

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The proof: Geometric Rényi Divergences

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Let $\mathcal{H}_{AC} := \mathcal{H}_A \otimes \mathcal{H}_C$ be a finite-dimensional Hilbert space. For $1 < \alpha < \infty$, and ρ_{AC}, σ_{AC} full-rank states, their *α-geometric Rényi divergence* is given by

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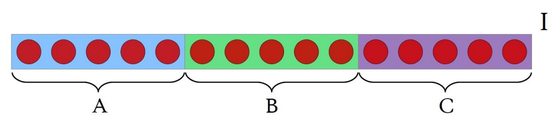

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In particular,

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If $\left\| \rho_A^{-1} \otimes \rho_C^{-1} \rho_{AC} - \mathbb{1}_{AC} \right\| \leq \varepsilon(\ell)$ $\Rightarrow$ Decay of mutual information.

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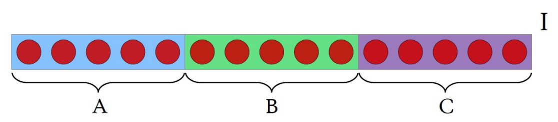

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The following chain of inequalities holds:

$$\frac{1}{2} \operatorname{Corr}(A:C)^2 \leq \frac{1}{2} \| \rho_{AC} - \rho_A \otimes \rho_C \|_1^2 \leq I_\rho(A:C)$$

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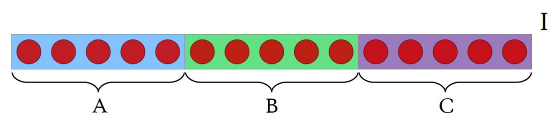

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Let $\Lambda \subset \mathbb{Z}$ and $H_\Lambda = \sum_{X \subset \Lambda} \Phi_X$ a finite range, local, non-commuting Hamiltonian. For a finite interval $I = XY \subset \mathbb{Z}$, let us write

$$E_{X,Y} := e^{-H_{XY}} e^{H_X + H_Y} .$$

Then, there is an absolute constant $\mathcal{G}$ such that:

(i) It holds:

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These and similar techniques are used repeatedly throughout the proof.

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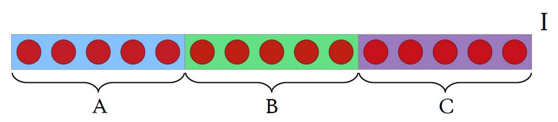
Local indistinguishability



Local indistinguishability (Bluhm-C.-Pérez Hernández, '21)

Consider the previous setting and $I = ABC \subset \mathbb{Z}$ with $|B| \geq 2\ell$. Assume exponential uniform clustering:

$$\mathrm{Corr}_{\rho_I}(A:C) \leq \varepsilon(\ell) \qquad \text{where} \qquad \rho_I = \frac{e^{-H_I}}{\mathrm{Tr}_I(e^{-H_I})} \,.$$

Then, for every pair of observables $Q_A \in \mathfrak{A}_A$ and $Q_C \in \mathfrak{A}_C$ we have

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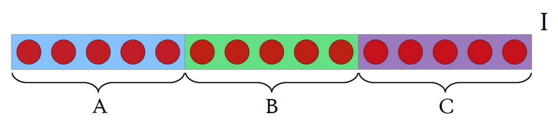

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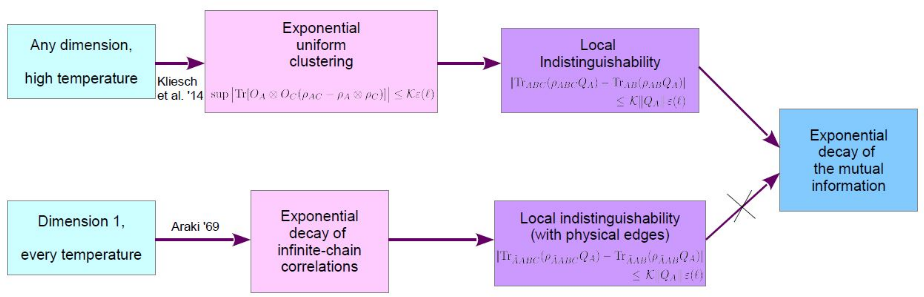

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For $\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$, two positive states ρ_{ABC}, σ_{ABC} such that $\sigma_{ABC} = \rho_{AB} \otimes \mathbb{1}_C / d_C$ and a $\mathcal{T} := \mathbb{1}_A / d_A \otimes \mathrm{Tr}_A$, we say that ρ_{ABC} is a *BS recoverable state* if

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