# The modified logarithmic Sobolev inequality for quantum spin systems

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Technical University of Munich

Munich Center for Quantum Science and Technology

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Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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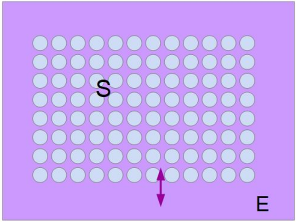

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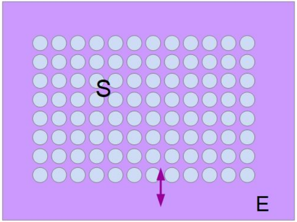


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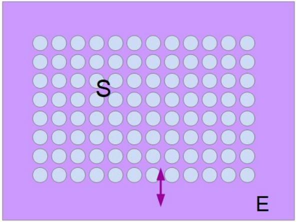


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A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t^*\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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We say that $\mathcal{L}_\Lambda^*$ satisfies **rapid mixing** if

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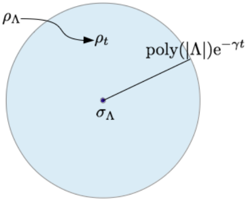

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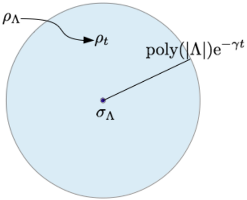

# Modified logarithmic Sobolev inequality

**Relative entropy:** $D(\rho\|\sigma) := \operatorname{tr}[\rho(\log\rho - \log\sigma)]$

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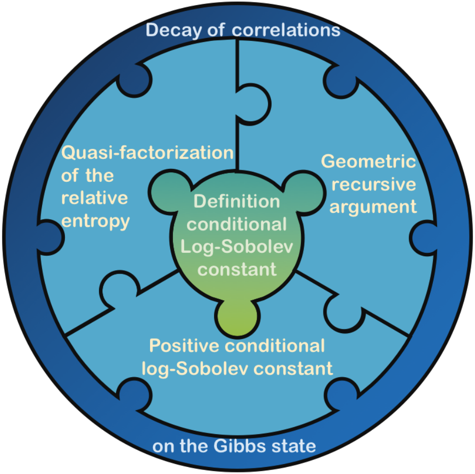
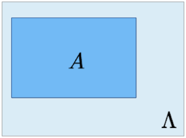
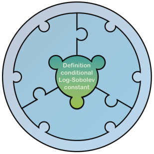
# Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



# Conditional MLSI constant

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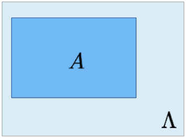


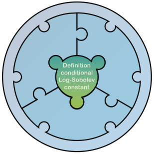


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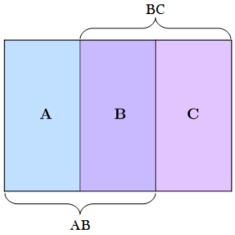
# Quasi-factorization of the relative entropy




## Quasi-factorization of the relative entropy

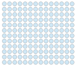
Given $\Lambda = ABC$, it is an inequality of the form:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[ D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on $\sigma_{ABC}$ and measures how far $\sigma_{AC}$ is from $\sigma_A \otimes \sigma_C$.

# Example: Tensor product fixed point

(C.-Lucia-Pérez García '18) $\mathcal{L}^*_\Lambda(\rho_\Lambda) = \sum_{x\in\Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$



$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

$$\leq \sum_{x\in\Lambda} D_x(\rho_\Lambda||\sigma_\Lambda)$$

$\alpha_\Lambda(\mathcal{L}^*_x) := \inf_{\rho_\Lambda\in\mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2D_x(\rho_\Lambda||\sigma_\Lambda)}$

$$\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}^*_x)}$$

$$\leq \frac{1}{2\inf_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}^*_x)} \sum_{x\in\Lambda} -\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]$$

$$= \frac{1}{2\inf_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}^*_x)} \left(-\operatorname{tr}[\mathcal{L}^*_\Lambda(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right)$$

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## MLSI, informal (C.-Rouzé-Stilck França '20)

Let $H_\Lambda$ be a local commuting Hamiltonian with $\beta < \beta_c$ and such that one of the following conditions holds:

1. $H_\Lambda$ is classical.
2. $H_\Lambda$ is a nearest neighbour Hamiltonian.
3. $\Lambda$ is 1D.

Then, there exists a local quantum Markov semigroup with fixed point $\sigma_\Lambda$, the Gibbs state of $H_\Lambda$, such that it has a positive **MLSI constant** which is independent of the system size.

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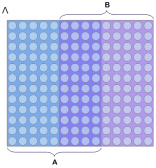


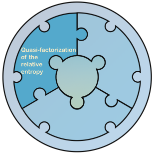


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For $A \subset \Lambda$, let $E_{A*} := \lim_{t\to\infty} \mathrm{e}^{t\mathcal{L}_A^*}$.

## Quasi-factorization via Pinching (Bardet-C.-Rouzé '20)

We have:

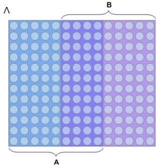
$$D(\rho \| E_{A\cup B*}(\rho)) \leq \frac{1}{1-2c_1} \left[ D(\rho\|E_{A*}(\rho)) + D(\rho\|E_{B*}(\rho)) \right] + \xi_{A^c \leftrightarrow B^c}(\rho) \,,$$

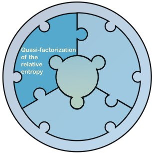
where

$$c_1 := \max_{\text{blocks}} \left\| E_A \circ E_B - E_{A\cup B} \right\| ,$$

and $\xi_{A^c \leftrightarrow B^c}(\rho)$ strongly depends on the Pinching.

# Quasi-factorization of the relative entropy

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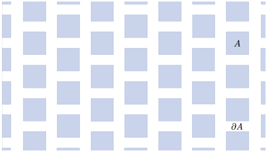


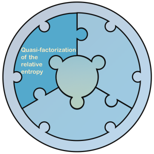


(Bravyi-Vyalyi '05) Nearest neighbour Schmidt semigroups.

Conditional expectation: Tiling $A$ + NN Schmidt semigroups $\Rightarrow \xi_{A^c \leftrightarrow B^c}(\rho) = 0$.

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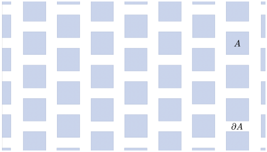


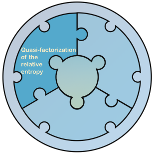


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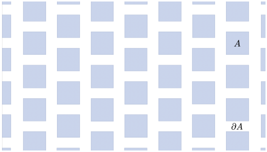


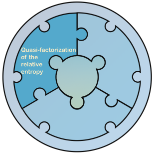


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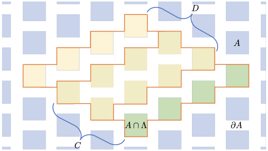


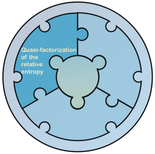


## Quasi-factorization (C.-Rouzé-Stilck França '20)

If $\omega := E_{A*}(\rho)$, for $C$ and $D$ as above,

$$D(\omega \| E_{C\cup D*}(\omega)) \leq \frac{1}{1-2c_1}\Big( D(\omega \| E_{C*}(\omega)) + D(\omega \| E_{D*}(\omega)) \Big)\,.$$

The Hamiltonian needs to be **classical, 1D or nearest neighbour**.

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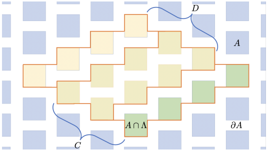


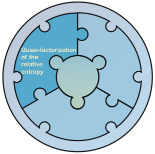


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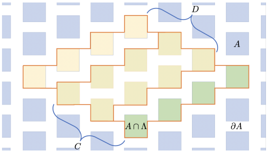


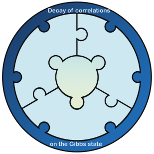


## Clustering of correlations

For high-enough temperature

$$c_1 = \max_{\text{blocks}} \| E_C \circ E_D - E_{C \cup D} \| \leq c \, |C \cup D| \, \mathrm{e}^{-\frac{\mathrm{d}(C \setminus D, D \setminus C)}{k}} ,$$

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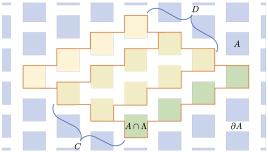


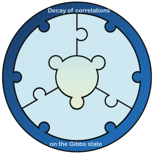


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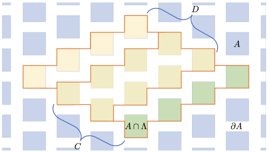


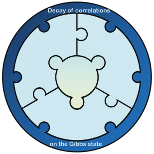


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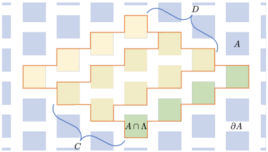


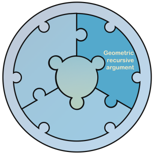


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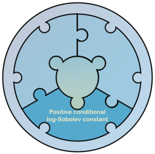


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Pinched MLSI + Positivity of the complete MLSI (Rouzé-Gao '21)

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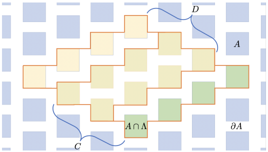


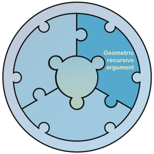


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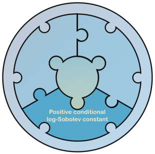


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