Logarithmic Sobolev Inequalities for Quantum Many-Body Systems

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Main topic of this talk

Field of study

Dissipative evolutions of quantum many-body systems

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Velocity of convergence of certain quantum dissipative evolutions to their thermal equilibriums.

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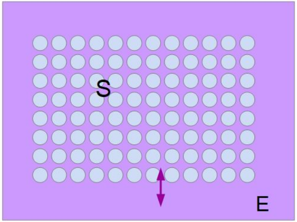

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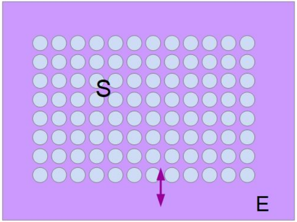

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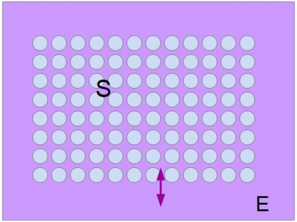

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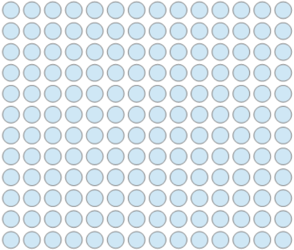

Figure: A quantum spin lattice system.

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- To every site $x \in \Lambda$ we associate $\mathcal{H}_x$ $(= \mathbb{C}^D)$.
- The global Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x\in\Lambda} \mathcal{H}_x$.
- The set of bounded linear endomorphisms on $\mathcal{H}_\Lambda$ is denoted by $\mathcal{B}_\Lambda := \mathcal{B}(\mathcal{H}_\Lambda)$.
- The set of density matrices is denoted by $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \,:\, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

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Physical evolution: $\rho \mapsto U\rho U^*$ $\rightsquigarrow$ Reversible

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The QMS studied is **reversible**, i.e. it satisfies **detailed balance** $\forall f, g \in \mathcal{A}$:

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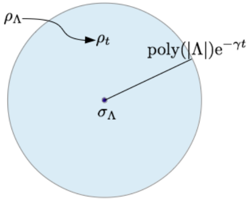

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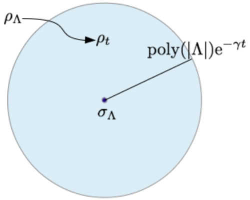

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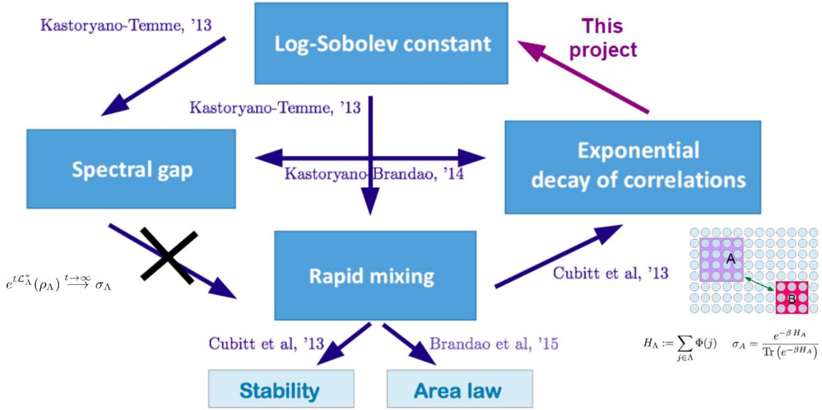

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(3) Decay of correlations on the Gibbs measure.

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(2) Recursive geometric argument.
Lower bound for the global log-Sobolev constant in terms of the log-Sobolev constant of a size-fixed region.

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(3) Decay of correlations on the Gibbs measure.

$\Downarrow$

Positive log-Sobolev constant.

Classical strategy

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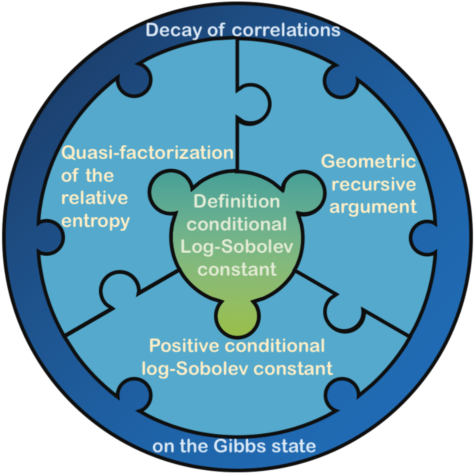
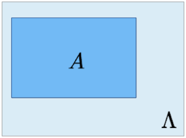
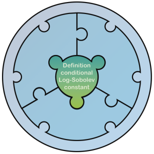
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Conditional MLSI constant





MLSI constant

The **MLSI constant** of $\mathcal{L}_\Lambda^*$ is defined by

$$\alpha(\mathcal{L}_\Lambda^*) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda^*(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

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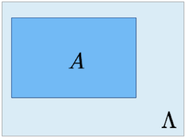

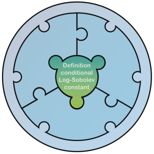

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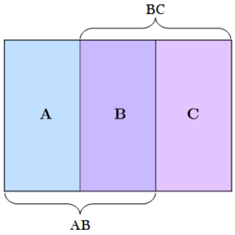
Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

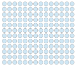
$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{D}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

Example: Tensor product fixed point

(C.-Lucia-Pérez García '18) $\mathcal{L}^*_\Lambda(\rho_\Lambda) = \sum_{x\in\Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda)$ **heat-bath**

$$D_x(\rho_\Lambda\|\sigma_\Lambda) := D(\rho_\Lambda\|\sigma_\Lambda) - D(\rho_{x^c}\|\sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x ,$$

$$D(\rho_\Lambda||\sigma_\Lambda) \leq$$

$$\leq \sum_{x\in\Lambda} D_x(\rho_\Lambda||\sigma_\Lambda)$$

$\alpha_\Lambda(\mathcal{L}^*_x) := \inf_{\rho_\Lambda\in\mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2D_x(\rho_\Lambda||\sigma_\Lambda)}$

$$\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}^*_x)}$$

$$\leq \frac{1}{2\inf_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}^*_x)} \sum_{x\in\Lambda} -\operatorname{tr}[\mathcal{L}^*_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]$$

$$= \frac{1}{2\inf_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}^*_x)} \left(-\operatorname{tr}[\mathcal{L}^*_\Lambda(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right)$$

$$\leq \left(-\operatorname{tr}[\mathcal{L}^*_\Lambda(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right) .$$

Dynamics

Let $\sigma_\Lambda = \frac{\mathrm{e}^{-\beta H_\Lambda}}{\mathrm{tr}\left[\mathrm{e}^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

$$\mathcal{L}_\Lambda^{H;*}(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right)$$

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The **Davies generator** is given by:

$$\mathcal{L}_\Lambda^D(X) := i[H_\Lambda, X] + \sum_{x\in\Lambda} \mathcal{L}_x^D(X),$$

where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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The **Schmidt generator** (Bravyi-Vyalyi '05) can be written as:

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Previous results

Let us recall: For $\alpha(\mathcal{L}_\Lambda^*)$ a MLSI constant,

$$\|\rho_t - \sigma_\Lambda\|_1 \leq \sqrt{2\log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda^*)\, t}.$$

Using the spectral gap $\lambda(\mathcal{L}_\Lambda^*)$:

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Let $\mathcal{L}_\Lambda^{H,D;*}$ be the **heat-bath** or **Davies** generator in 1D. Then, $\mathcal{L}_\Lambda^{H,D;*}$ has a positive spectral gap that is independent of the system size, for every temperature.

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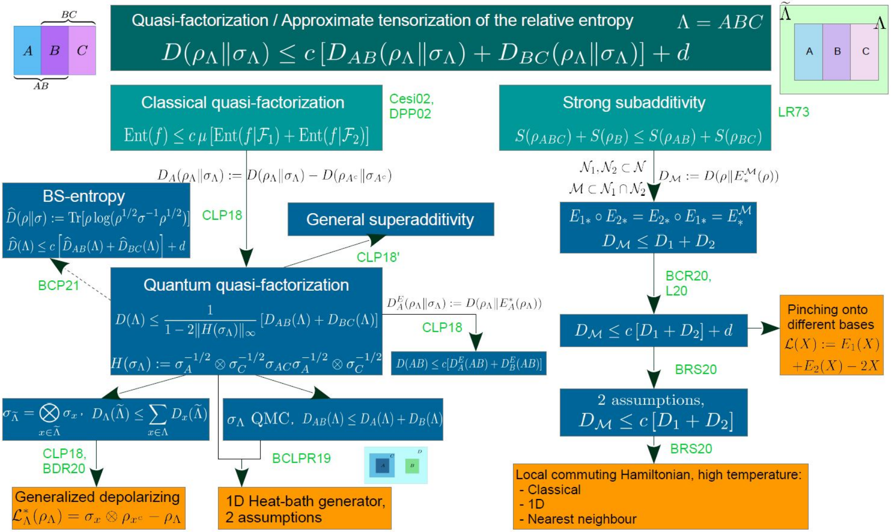
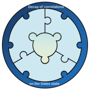


Some results



$$D_B(\rho_\Lambda || \sigma_\Lambda) = D(\rho_\Lambda || \sigma_\Lambda) - D(\rho_{B^c} || \sigma_{B^c}).$$

Assumption 1

In a tripartite Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_C \otimes \mathcal{H}_B$, A and B not connected, we have

$$\left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty \leq K < \frac{1}{2}.$$

In particular, Gibbs states at high enough temperature satisfy this.

Assumption 2

For any $B \subset \Lambda$, $B = B_1 \cup B_2$, it holds:

$$D_B(\rho_\Lambda || \sigma_\Lambda) \leq f(\sigma_{B\partial}) \left(D_{B_1}(\rho_\Lambda || \sigma_\Lambda) + D_{B_2}(\rho_\Lambda || \sigma_\Lambda) \right).$$

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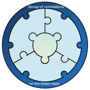

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In 1D, if Assumptions 1 and 2 hold, for a k-local commuting Hamiltonian, the **heat-bath** dynamics has a positive MLSI constant.

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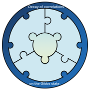

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Previous results

MLSI for Schmidt (C.-Rouzé-Stilck França '20)

Let H_Λ be a local commuting Hamiltonian with $\beta < \beta_c$ and such that one of the following conditions holds:

1. H_Λ is classical.
2. H_Λ is a nearest neighbour Hamiltonian.
3. Λ is 1D.

Then, there exists a local quantum Markov semigroup with fixed point $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}[e^{-\beta H_\Lambda}]}$, the Gibbs state of H_Λ, such that it has a positive **MLSI constant** which is independent of the system size.

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MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '21)

Let $\mathcal{L}_\Lambda^{D;*}$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^{D;*}$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^{D;*}) = \Omega(\ln(|\Lambda|)^{-1})$.

Rapid mixing:

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}.$$

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In the setting above, $\mathcal{L}_\Lambda^{D;*}$ has rapid mixing.

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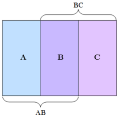
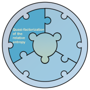
Proof: Conditional relative entropies + Quasi-factorization





Conditional relative entropies: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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Heat-bath cond. expectation: $E^*_A(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2} \sigma_{A^c}^{-1/2} \operatorname{tr}_A[\,\cdot\,] \, \sigma_{A^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n$.

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$$D(\rho_{ABC} \| \sigma_{ABC}) \leq \xi(\sigma_{AC}) \left[D_{AB}(\rho_{ABC} \| \sigma_{ABC}) + D_{BC}(\rho_{ABC} \| \sigma_{ABC}) \right],$$

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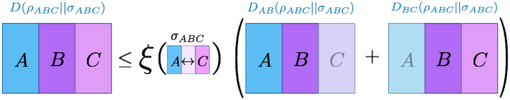

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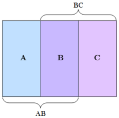

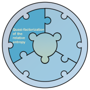

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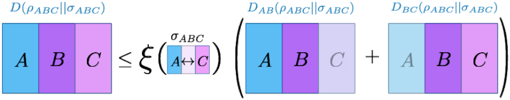
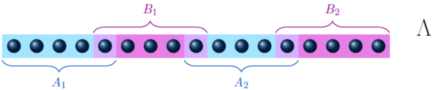
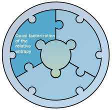
$$\xi(\sigma_{AC}) = \frac{1}{1 - 2 \left\| \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} \, \sigma_{AC} \, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC} \right\|_\infty}.$$



PROOF: QUASI-FACTORIZATION





$\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\mathrm{tr}(e^{-\beta H_\Lambda})}$ is the Gibbs state of a k-local, commuting Hamiltonian H_Λ.

QUASI-FACTORIZATION

Let $A \cup B = \Lambda \subset \mathbb{Z}$ and $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}_\Lambda$. The following holds

$$D(\rho_\Lambda || \sigma_\Lambda) \leq \xi(\sigma_{A^c B^c}) \left[D_A(\rho_\Lambda || \sigma_\Lambda) + D_B(\rho_\Lambda || \sigma_\Lambda) \right],$$

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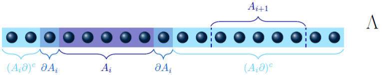
QUASI-FACTORIZATION FOR QUANTUM MARKOV CHAINS (Bardet-C.-Lucia-Pérez García-Rouzé'19)

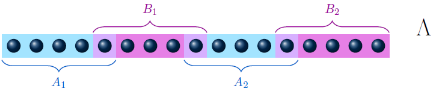
Since σ_Λ is a QMC between $A_i \leftrightarrow \partial(A_i) \leftrightarrow (A_i \cup \partial A_i)^c$, then:

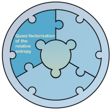
$$D_A(\rho_\Lambda || \sigma_\Lambda) \leq \sum_i D_{A_i}(\rho_\Lambda || \sigma_\Lambda).$$

$$\sigma_\Lambda = \bigoplus_{j \in J} \sigma_{A_i (\partial a_i)_j^L} \otimes \sigma_{(\partial a_i)_j^R (A_i \cup \partial A_i)^c}$$



Proof: Quasi-factorization





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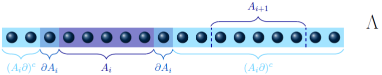
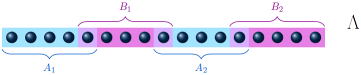
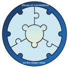


Proof: Decay of correlations





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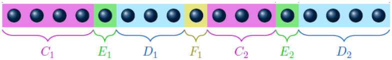

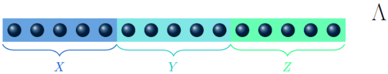

Decay of correlations, (Bluhm-C.-Pérez Hernández, '21)

Let σ_{XYZ} be the Gibbs state of a finite-range, translation-invariant Hamiltonian. There is $\ell \mapsto \delta(\ell)$ with exponential decay such that:

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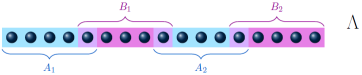

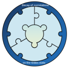

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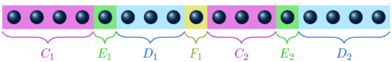

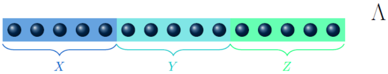

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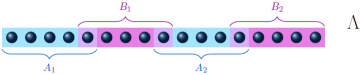

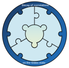

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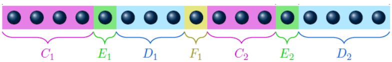

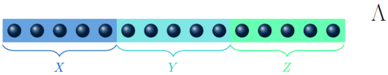

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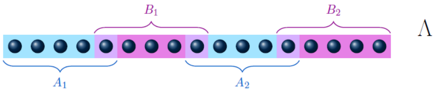

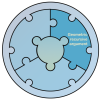

Let us recall: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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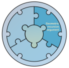
Comparison between conditional relative entropies (Bardet-C.-Rouzé, '20)

$$D_A(\rho_\Lambda \| \sigma_\Lambda) \leq D_A^E(\rho_\Lambda \| \sigma_\Lambda)$$

Therefore, by this and  +  , we have:

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and thus

$$\alpha(\mathcal{L}_\Lambda^{H;*}) \geq \frac{K}{\xi(\sigma_{A^c B^c})} \min \left\{ \alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}), \alpha_{B_i}(\mathcal{L}_\Lambda^{H;*}) \right\} ,$$

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$$\alpha_{A_i}(\mathcal{L}_\Lambda^{H;*}) = \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}\left[\mathcal{L}_{A_i}^{H;*}(\rho_\Lambda)(\ln \rho_\Lambda - \ln \sigma_\Lambda) \right]}{D(\rho_\Lambda \| E_{A_i}^*(\rho_\Lambda))} .$$

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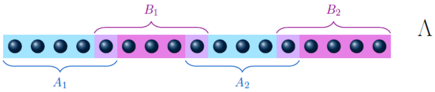

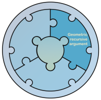

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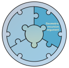
Therefore, by this and  +  , we have:

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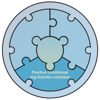
Proof: Positive CMLSI



Reduction of conditional relative entropies (Gao-Rouzé, '21)

$$D(\rho_\Lambda \| E^*_{A_i}(\rho_\Lambda)) \leq 4 k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E^*_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda},$$

where $\lambda < 1$ is a constant related to the spectral gap by the detectability lemma.

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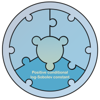

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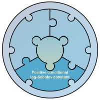

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The CMLSI of the local generators is positive:

$$\alpha_c(\mathcal{L}_j^{D;*}) := \inf_{k \in \mathbb{N}} \alpha(\mathcal{L}_j^{D;*} \otimes \mathrm{Id}_k) > 0 \, .$$

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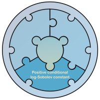

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Last step

Heat-bath cond. expectation: $E_A^{H;*}(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2} \sigma_{A^c}^{-1/2} \operatorname{tr}_A[\,\cdot\,]\, \sigma_{A^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n .$

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