Towards a unification of different measures of correlation and locality in Gibbs states

Angela Capel (University of Tuebingen)

Based on

From decay of correlations to locality and stability of the Gibbs state

arXiv:2310.09182

Joint work with

Massimo Moscolari
(Politecnico di Milano)

Stefan Teufel
(U. Tuebingen)

Tom Wessel
(U. Tuebingen)

Strong decay of correlations for Gibbs states in any dimension

arXiv:2401.10147

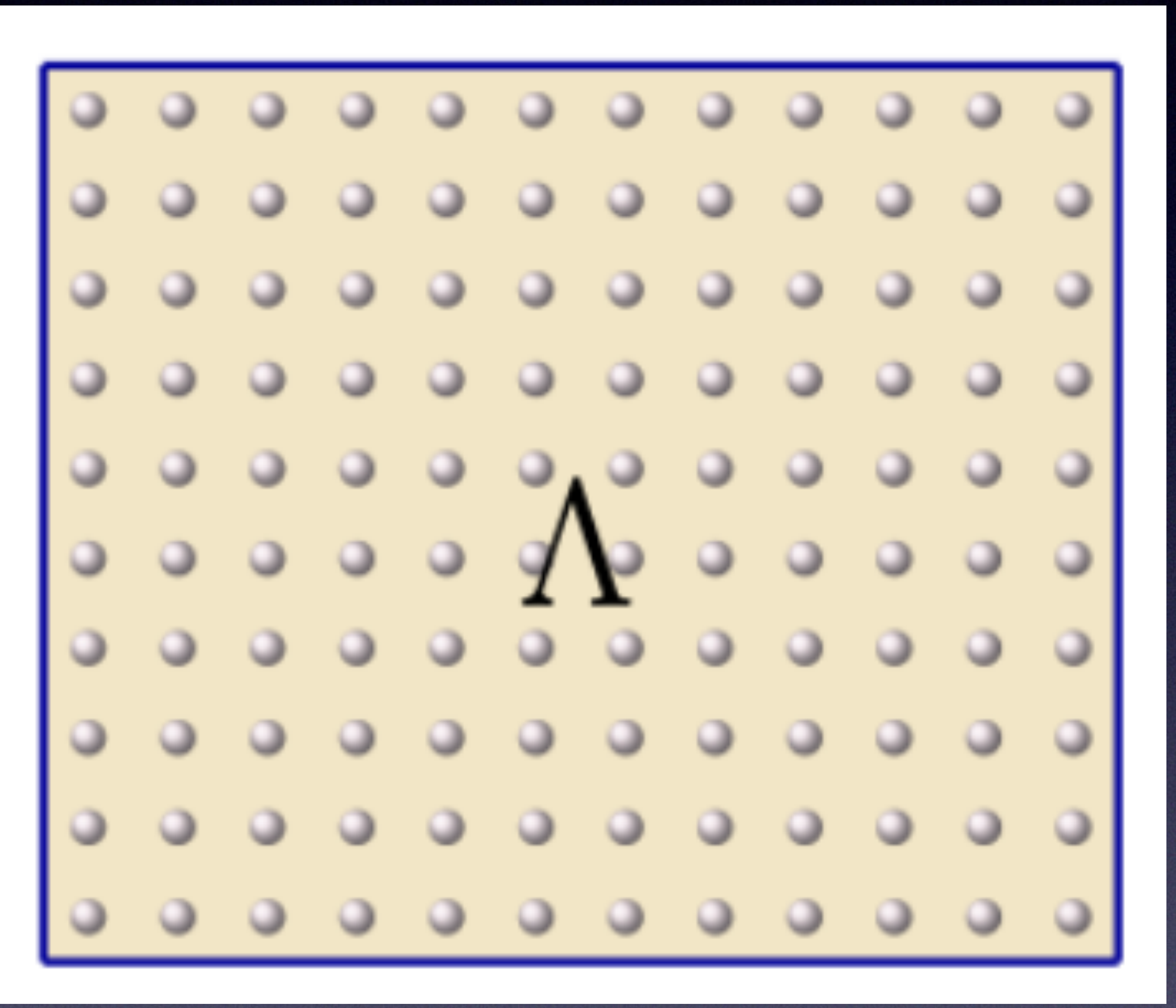
Joint work with

Andreas Bluhm
(U. Grenoble Alpes)

Antonio Pérez-Hernández
(UNED, Spain)

Introduction to the setting

Study of locality, stability and correlations on quantum Gibbs states

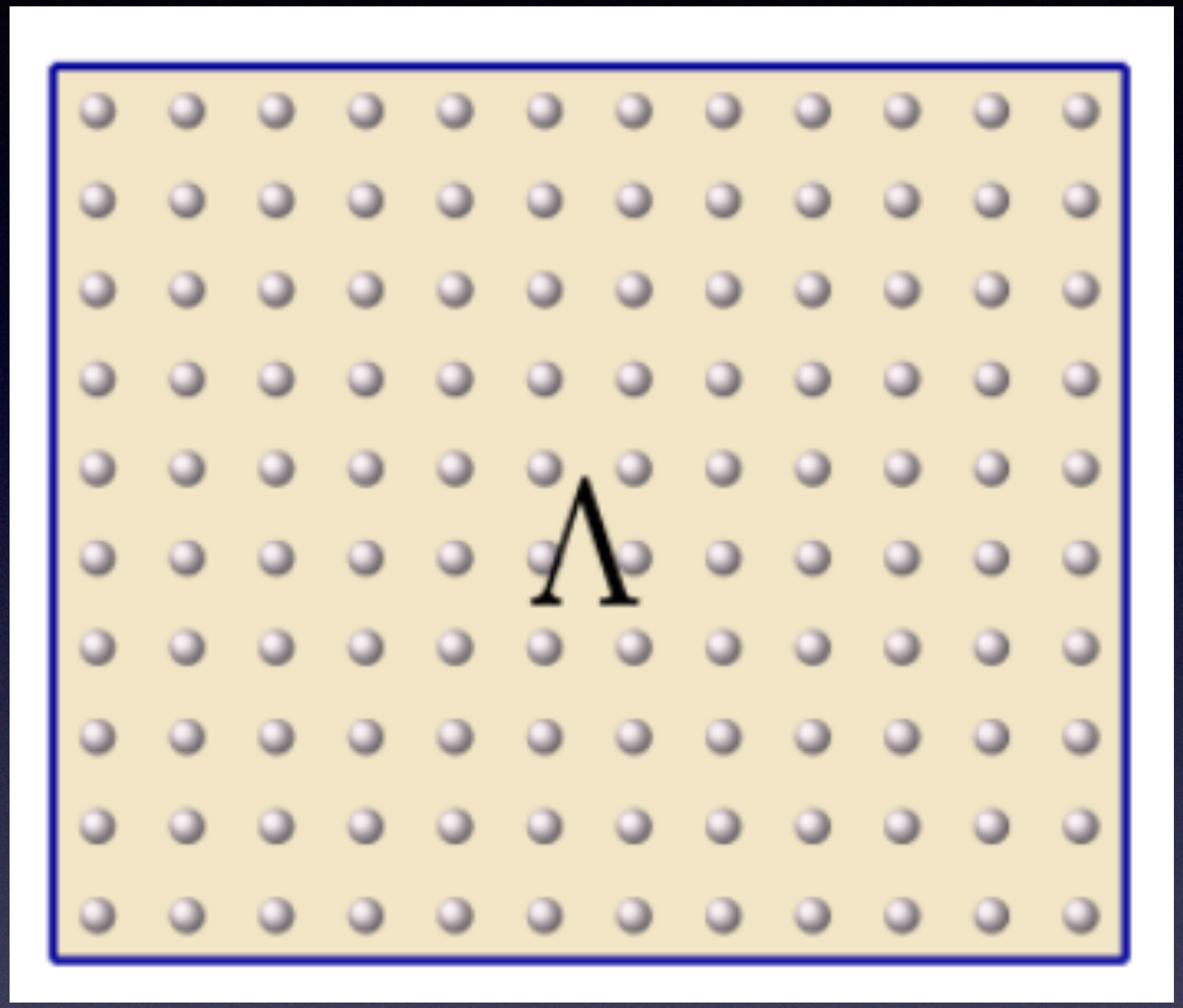


- Lattice $\Lambda \subset\subset \mathbb{Z}^D$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$
    - Short-range (exponentially-decaying interactions)
    - Finite-range (k-local interactions)
- Gibbs state (at inverse temperature $\beta > 0$) $\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$

Properties:

- Locality (from systems to states)
- Stability (against perturbations)
- Correlations (decay between spatially separated regions)

SHORT-RANGE HAMILTONIANS

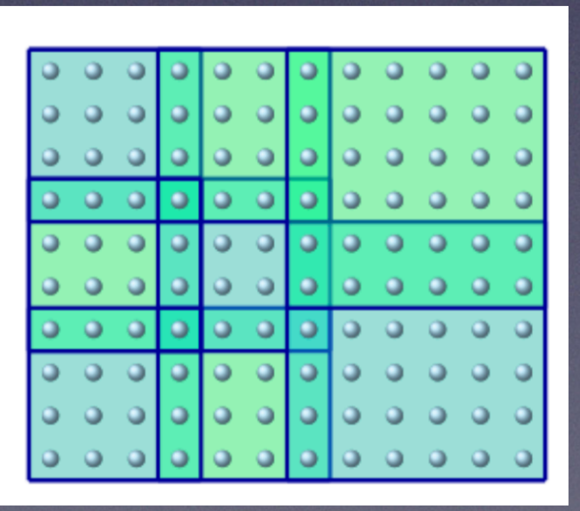


- Lattice $\Lambda \subset\subset \mathbb{Z}^D$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$
  - Short-range (exponentially-decaying interactions)

$$\|H_\Lambda\|_{\lambda,\mu} := \sup_{x \in V} \sum_{X \ni x} \|H_X\| e^{\lambda |X| + \mu \mathrm{diam}(X)} < \infty$$

  - Finite-range (k-local interactions)

$$H_X = 0 \text{ for } \mathrm{diam}(X) > k \text{ and } \|H_X\| < J \;\; \forall X \subset \Lambda$$



- Gibbs state (at inverse temperature $\beta > 0$)

$$\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$$

Properties of Short-range Hamiltonians

Main goal of this talk:

"Unification of several properties of locality and measures of correlations on Gibbs states"

Summary of main results:

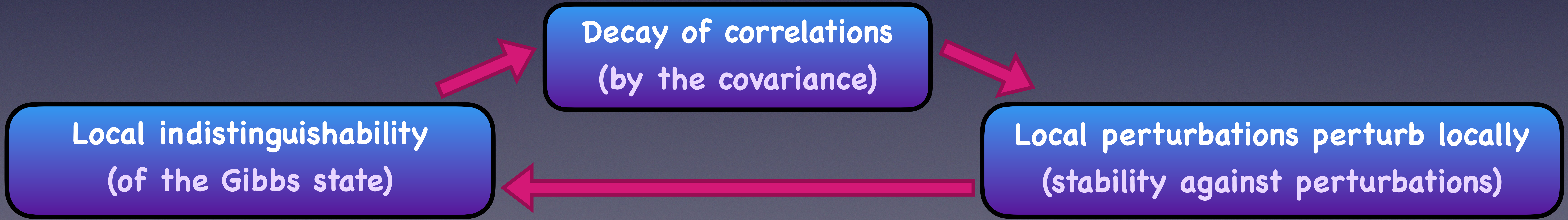
- From decay of correlations to locality and stability of the Gibbs state

Decay of correlations (by the covariance) → **Local perturbations perturb locally** (stability against perturbations) → **Local indistinguishability** (of the Gibbs state) → **Decay of correlations**

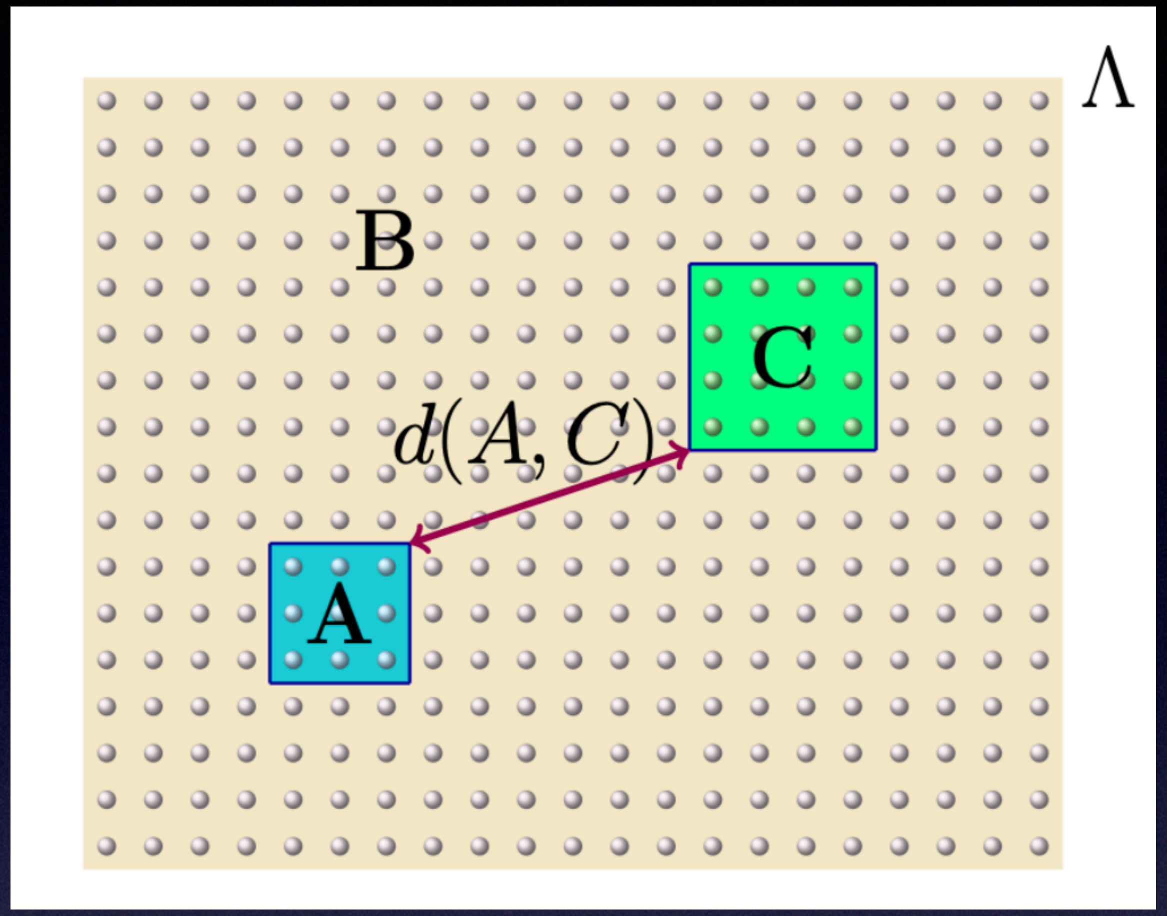
- Strong decay of correlations for Gibbs states in any dimension

Effective Hamiltonian → **Equivalence of several measures of correlations** (covariance, mutual information, mixing condition)

From decay of correlations to locality and stability of the Gibbs state (C., Moscolari, Teufel, Wessel, '23)



Decay of correlations (Covariance)



- Lattice $\Lambda \subset\subset \mathbb{Z}^D$, $\Lambda = ABC$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$
- Gibbs state (at inverse temperature $\beta > 0$) $\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$
- Family of Hamiltonians $(H_\Lambda)_{\Lambda \subset\subset \mathbb{Z}^D}$

Covariance

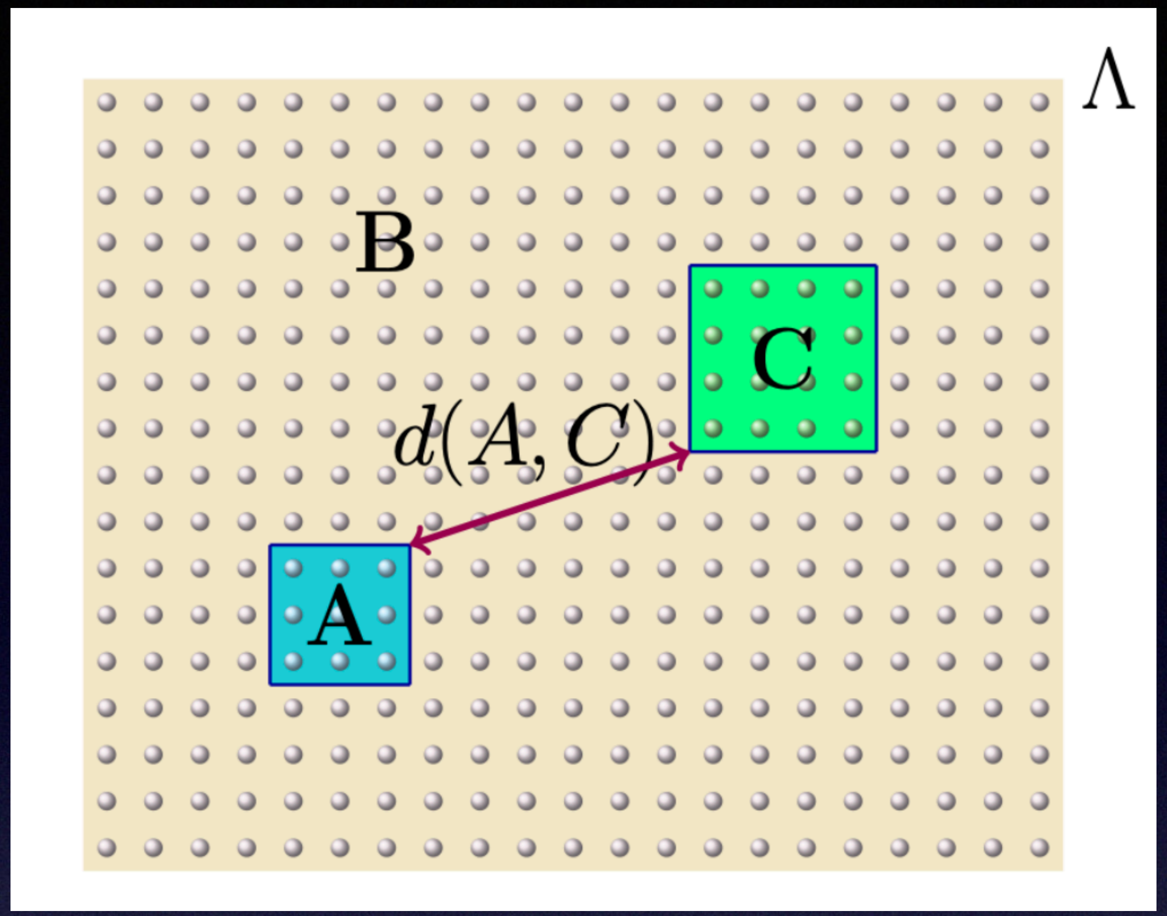
$$\mathrm{Cov}_{\rho^\Lambda}(A, C) = \sup_{\|O_A\|=\|O_C\|=1} |\mathrm{Tr}[\rho^\Lambda O_A O_C] - \mathrm{Tr}[\rho^\Lambda O_A]\mathrm{Tr}[\rho^\Lambda O_C]|$$

Exponential decay with $d(A, C)$?

$$\mathrm{Cov}_{\rho^\Lambda}(A, C) \leq K_{Cov} f_{Cov}(A, C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$

(for $K_{Cov} > 0$, $\alpha_{Cov} > 0$ and $f_{Cov}(A, C)$ depending on the size of A and C, but uniform in Λ)

DECAY OF CORRELATIONS (COVARIANCE)

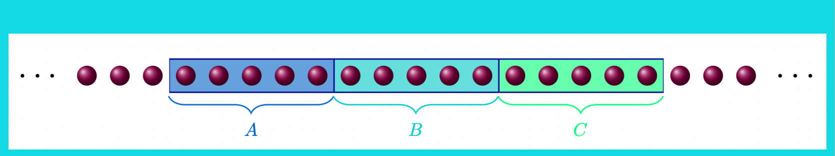
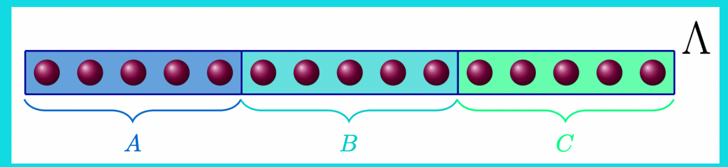


- Lattice $\Lambda \subset\subset \mathbb{Z}^D, \quad \Lambda = ABC$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$, family of Hamiltonians $(H_\Lambda)_{\Lambda \subset\subset \mathbb{Z}^D}$
- Gibbs state $\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$

Exponential decay covariance?

$$\mathrm{Cov}_{\rho^\Lambda}(A, C) = \sup_{\|O_A\|=\|O_C\|=1} |\mathrm{Tr}[\rho^\Lambda O_A O_C] - \mathrm{Tr}[\rho^\Lambda O_A]\mathrm{Tr}[\rho^\Lambda O_C]| \leq K_{Cov} f_{Cov}(A, C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$

1D, translation-invariant Hamiltonian

	Infinite chain 	Finite chain 
Finite range	$\beta > 0$ [Araki, '69]	$\beta > 0$ [Bluhm, C. Pérez-Hernández, '22]
Short range	$\beta < \beta_1$ [Pérez-García, Pérez-Hernández, '23]	$\beta < \beta_1$ [C., Moscolari, Teufel, Wessel, '23]

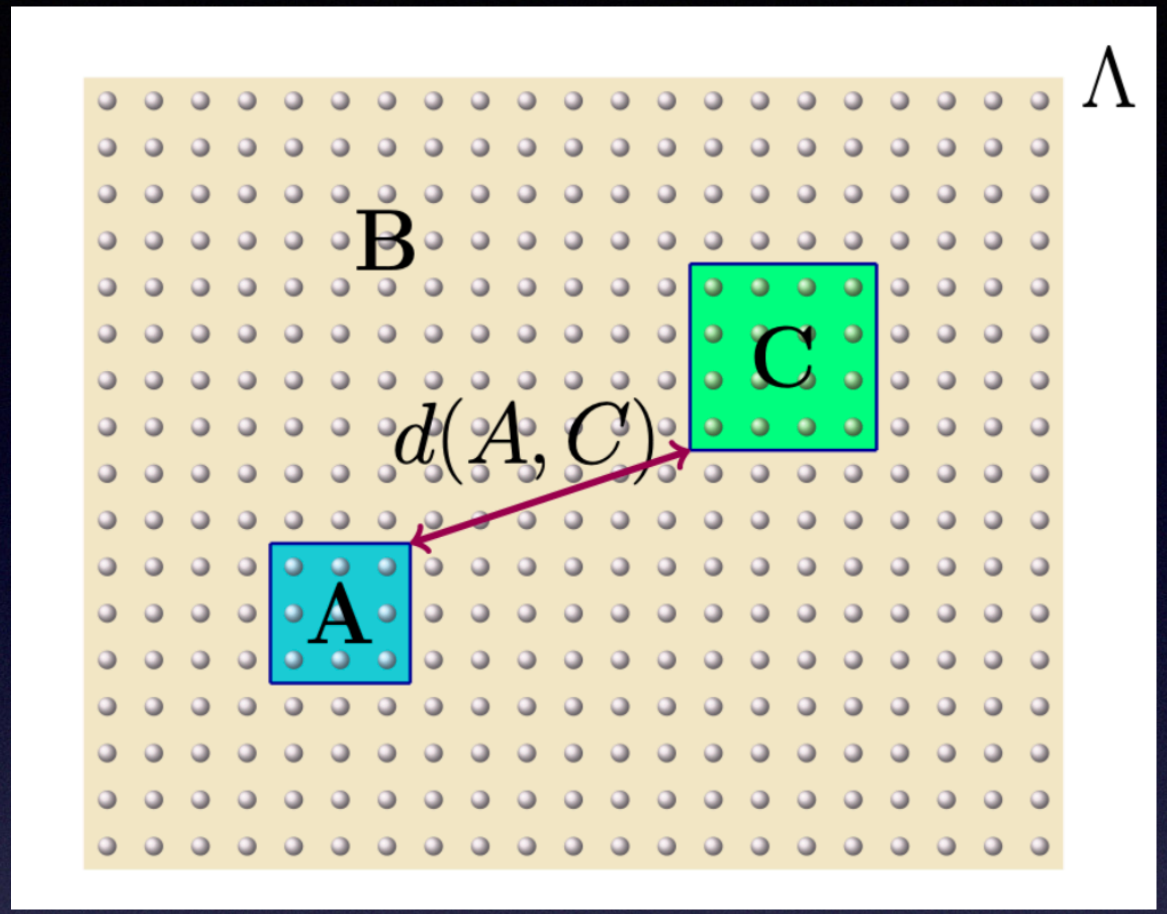
$\beta_1 \to \infty$ for finite range, $f_{Cov}(A, C) = 1$

High D, high-enough temperature

$$\beta < \beta_*$$

Finite range	[Kliesch et al., '14] $f_{Cov}(A, C) = \mathcal{O}(\lvert\partial A\rvert, \lvert\partial C\rvert)$
Short range	[Fröhlich-Ueltschi, '15] $f_{Cov}(A, C) = \mathcal{O}(\lvert A\rvert \lvert C\rvert)$

Local Perturbations Perturb Locally (LPPL)



- Lattice $\Lambda \subset\subset \mathbb{Z}^D, \quad \Lambda = ABC$
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  - Gibbs state $\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$
- Perturbation V_C , observable O_A
  - Perturbed Gibbs state $\widetilde{\rho}^\Lambda := \frac{e^{-\beta(H_\Lambda+V_C)}}{\mathrm{Tr}[e^{-\beta(H_\Lambda+V_C)}]}$

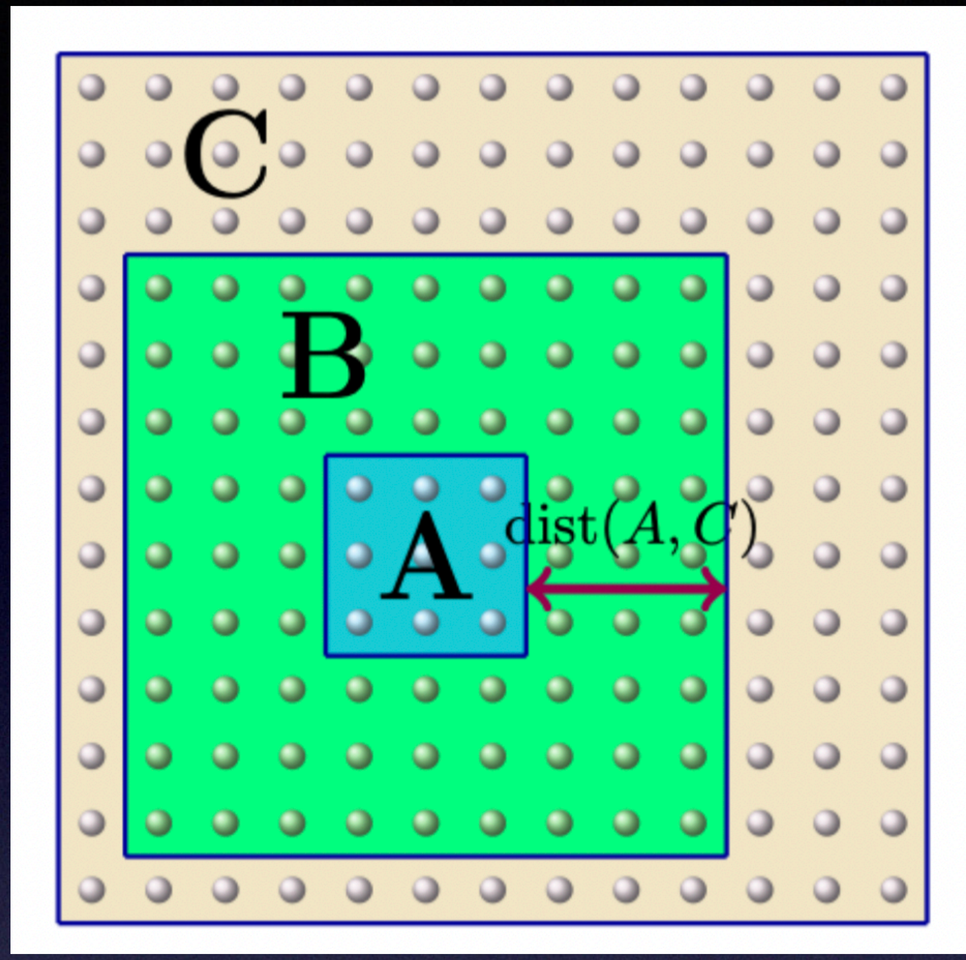
Local Perturbations Perturb Locally (LPPL)

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda\, O_A) - \mathrm{Tr}_{ABC}(\widetilde{\rho}^\Lambda\, O_A)\right| \leq K_{LPPL}\|O_A\|\, f_{LPPL}(A,C)\, \mathrm{e}^{c\beta\|V_C\|}\, \mathrm{e}^{-\alpha_{LPPL} d(A,C)}$$

(for $K_{LPPL} > 0$, $\alpha_{LPPL} > 0$ and $f_{LPPL}(A,C)$ depending on the size of A and C, but uniform in Λ)

LPPL is frequently used for locality of ground states [Bachmann et al., '11] [de Roeck, Schütz, '15] [Henheik et al., '22]

Local Indistinguishability



- Lattice $\Lambda \subset\subset \mathbb{Z}^D, \quad \Lambda = ABC$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$, family of Hamiltonians $(H_\Lambda)_{\Lambda \subset\subset \mathbb{Z}^D}$
  - Gibbs state $\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$
  - Gibbs state (of truncated Hamiltonian) $\rho^{AB} := \frac{e^{-\beta H_{AB}}}{\mathrm{Tr}[e^{-\beta H_{AB}}]}$

Local indistinguishability

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{AB}(\rho^{AB} O_A)\right| \leq K_{LI} \|O_A\| f_{LI}(A,C)\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$$

(for $K_{LI} > 0$, $\alpha_{LI} > 0$ and $f_{LI}(A,C)$ depending on the size of A and C, but uniform in Λ)

Previously used in [Brandao-Kastoryano, '19] for efficient preparation of Gibbs states

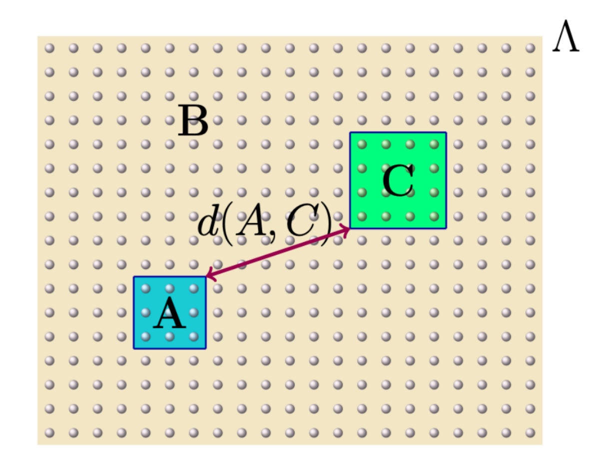
Also used in [Bluhm, C., Pérez-Hernández, '22] [Onorati et al., '23]

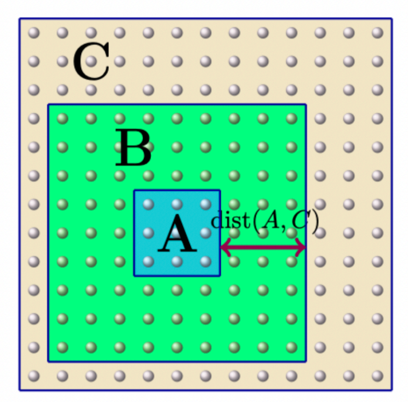
CONNECTIONS IN HIGH DIMENSION

In [C., Moscolari, Teufel, Wessel, '23], for short-range interactions:

Exponential decay covariance

$$\mathrm{Cov}_{\rho^\Lambda}(A, C) \le K_{Cov} f_{Cov}(A, C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$





Local indistinguishability

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{AB}(\rho^{AB} O_A)\right| \le K_{LI} \|O_A\| f_{LI}(A, C)\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$$

Local Perturbations Perturb Locally (LPPL)

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{ABC}(\widetilde{\rho}^\Lambda O_A)\right| \le K_{LPPL} \|O_A\| f_{LPPL}(A, C)\, \mathrm{e}^{c\beta \|V_C\|}\, \mathrm{e}^{-\alpha_{LPPL} d(A,C)}$$

Previous result: In [Brandao-Kastoryano, '19] exponential decay of covariance implies local indistinguishability for finite-range interactions

Connections in High Dimension

High dimension

$$\beta < \beta_*$$

[Kliesch et al., '14] finite-range

[Fröhlich-Ueltschi, '15] short-range

In [C., Moscolari, Teufel, Wessel, '23], for short-range interactions:

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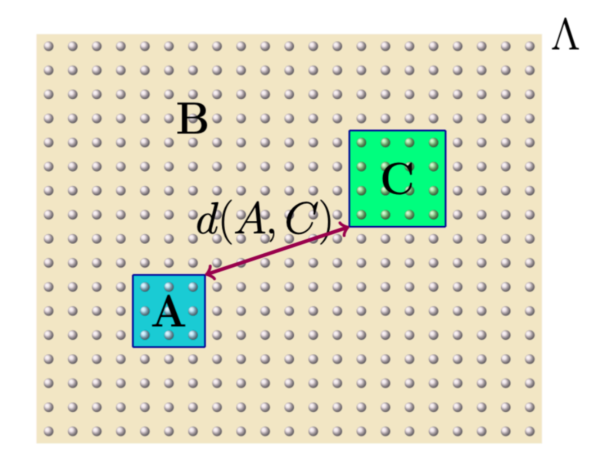

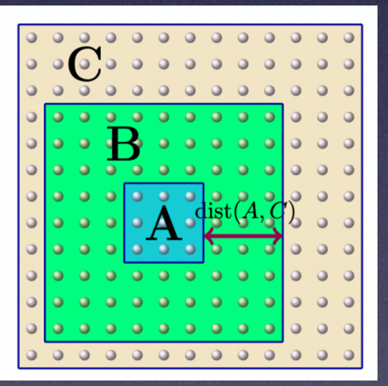

Local indistinguishability

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{AB}(\rho^{AB} O_A)\right| \le K_{LI} \|O_A\| f_{LI}(A,C)\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$$

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Connections in 1D, translation-invariant

1D, translation-invariant

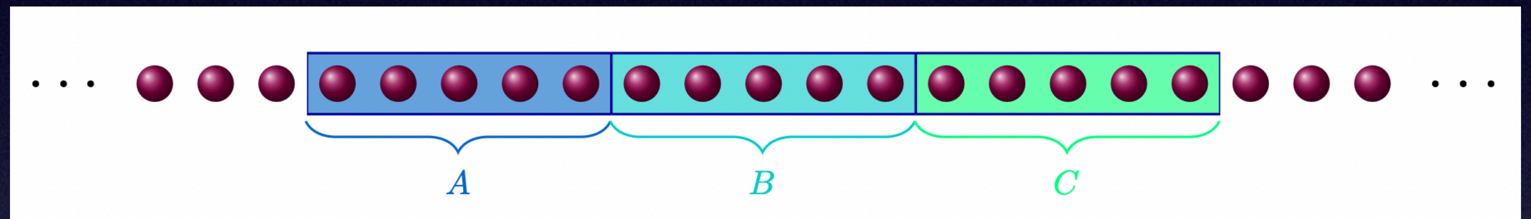
$\beta < \beta_1$

$\beta_1 \to \infty$ for finite range,

[Araki, '69] finite-range ↓ [Pérez-García, Pérez-Hernández '23] short-range

Exponential decay covariance (infinite chain)

$$\mathrm{Cov}_{\rho^\Lambda}(A, C) \leq K_{Cov}\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$



[Bluhm, C., Pérez-Hernández, '22] finite-range ↓ [C., Moscolari, Teufel, Wessel, '23] short-range

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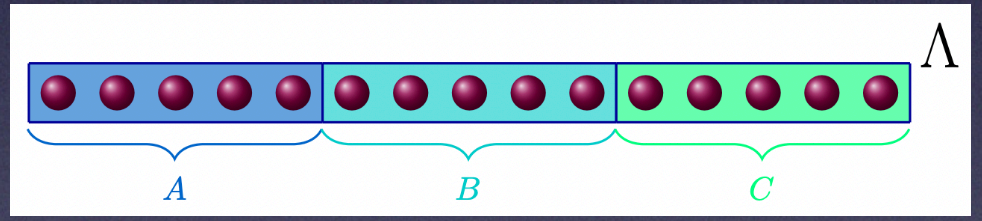
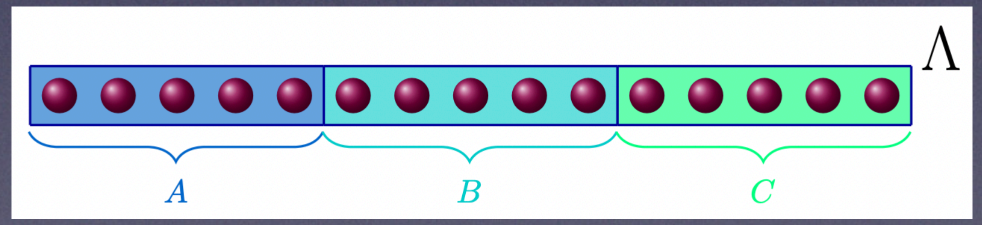
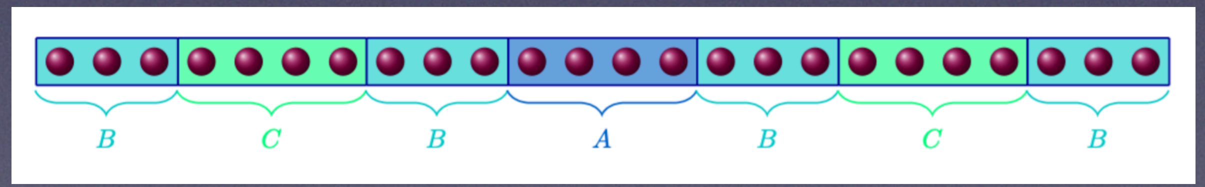




[C., Moscolari, Teufel, Wessel, '23]



Local indistinguishability

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{AB}(\rho^{AB} O_A)\right| \leq K_{LI} \|O_A\|\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$$

Local Perturbations Perturb Locally (LPPL)

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{ABC}(\widetilde{\rho}^\Lambda O_A)\right| \leq K_{LPPL} \|O_A\|\, \mathrm{e}^{3\beta \|V_C\|}\, \mathrm{e}^{-\alpha_{LPPL} d(A,C)}$$

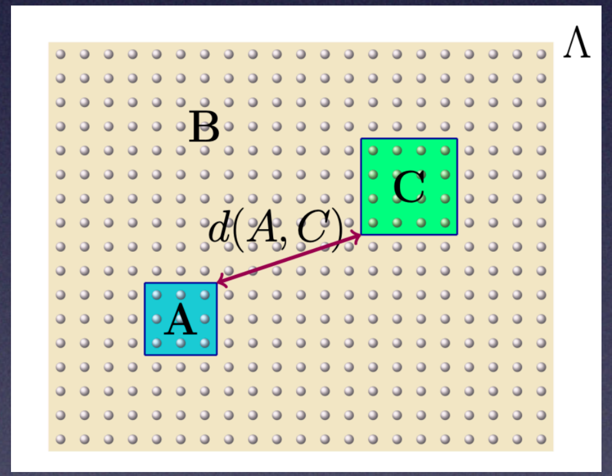
Proof in High Dimension

Main ingredients:

- Quantum Belief Propagation (QBP) [Hastings, '07], [Kim, '12] and many more
- Lieb-Robinson bounds [Lieb-Robinson, '72]

Exponential decay covariance

$$\mathrm{Cov}_{\rho^\Lambda}(A, C) \leq K_{Cov} f_{Cov}(A, C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$



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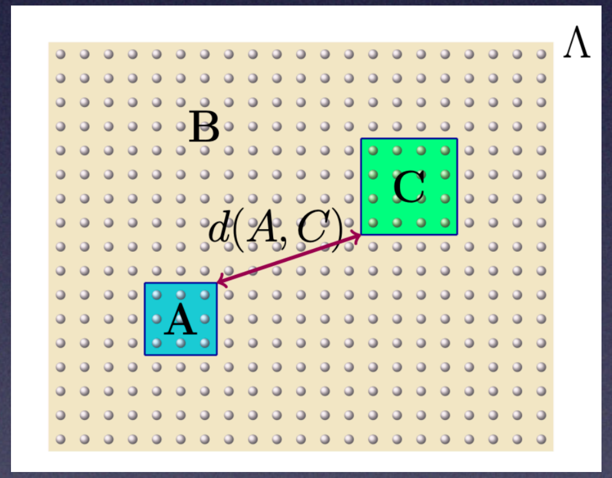

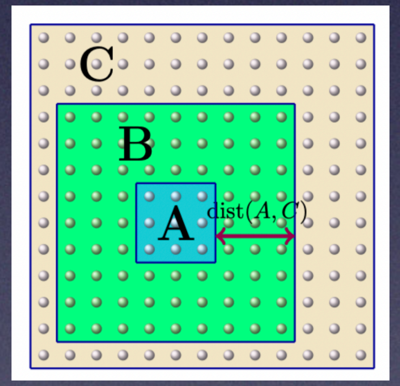

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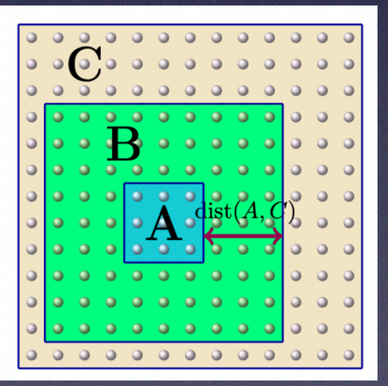

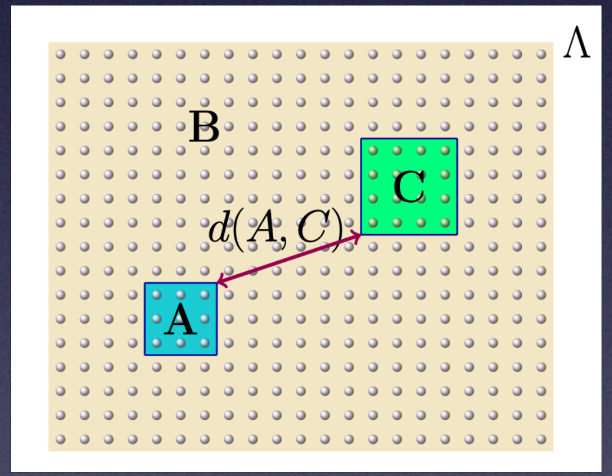

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Quantum Belief Propagation

- Lattice $\Lambda \subset\subset \mathbb{Z}^D, \quad \Lambda = ABC$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$, perturbation V_C

Quantum Belief Propagation

$$e^{-\beta(H_\Lambda + V_C)} = \eta(V_C) e^{-\beta H_\Lambda} \eta(V_C)^*$$

with $\eta(V) = \sum_{m=0}^{\infty} \left(\frac{-\beta}{2}\right)^m \int_0^1 ds_1 \int_0^{s_1} ds_2 \dots \int_0^{s_{m-1}} ds_m \, \mho^{s_n}(V) \cdot \ldots \cdot \mho^{s_1}(V)$ for $\mho_H^s(V) := \int_{\mathbb{R}} dt \, f_\beta(t) \, e^{-iH(s)t} V e^{iH(s)t}$

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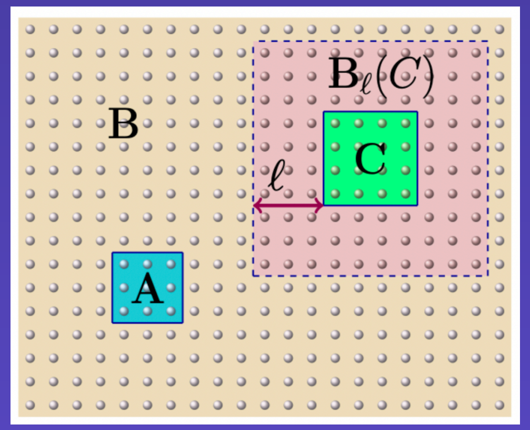
Quantum Belief Propagation for short-range interactions [C., Moscolari, Teufel, Wessel, '23]

Path of Hamiltonians $H_\Lambda(s) := H_\Lambda + sV_C$

- For $H_\Lambda(s)$ we have $e^{-\beta H_\Lambda(s)} = \eta(s) e^{-\beta H_\Lambda} \eta(s)^*$, $\quad \|\eta(s)\| \leq e^{\frac{\beta}{2} s \|V_C\|}$
- For the Gibbs states $\rho^\Lambda(s) := \dfrac{e^{-\beta H_\Lambda(s)}}{\mathrm{Tr}[e^{-\beta H_\Lambda(s)}]}$ we have $\rho^\Lambda_\beta(s) = \tilde{\eta}(s) \rho^\Lambda_\beta(0) \tilde{\eta}(s)^*$, $\quad \|\tilde{\eta}(s)\| \leq e^{\beta s \|V_C\|}$

  with $\|\rho^\Lambda_\beta(s) - \rho^\Lambda_\beta(0)\|_1 \leq e^{2\beta s \|V_C\|} - 1$



- Moreover $\|\tilde{\eta}(s) - \tilde{\eta}_\ell(s)\| \leq \kappa \beta s \|V_C\| e^{\beta s \|V_C\|} e^{-\gamma \ell}$ for $\eta_\ell(s)$ supported in $B_\ell(C)$

Site removal

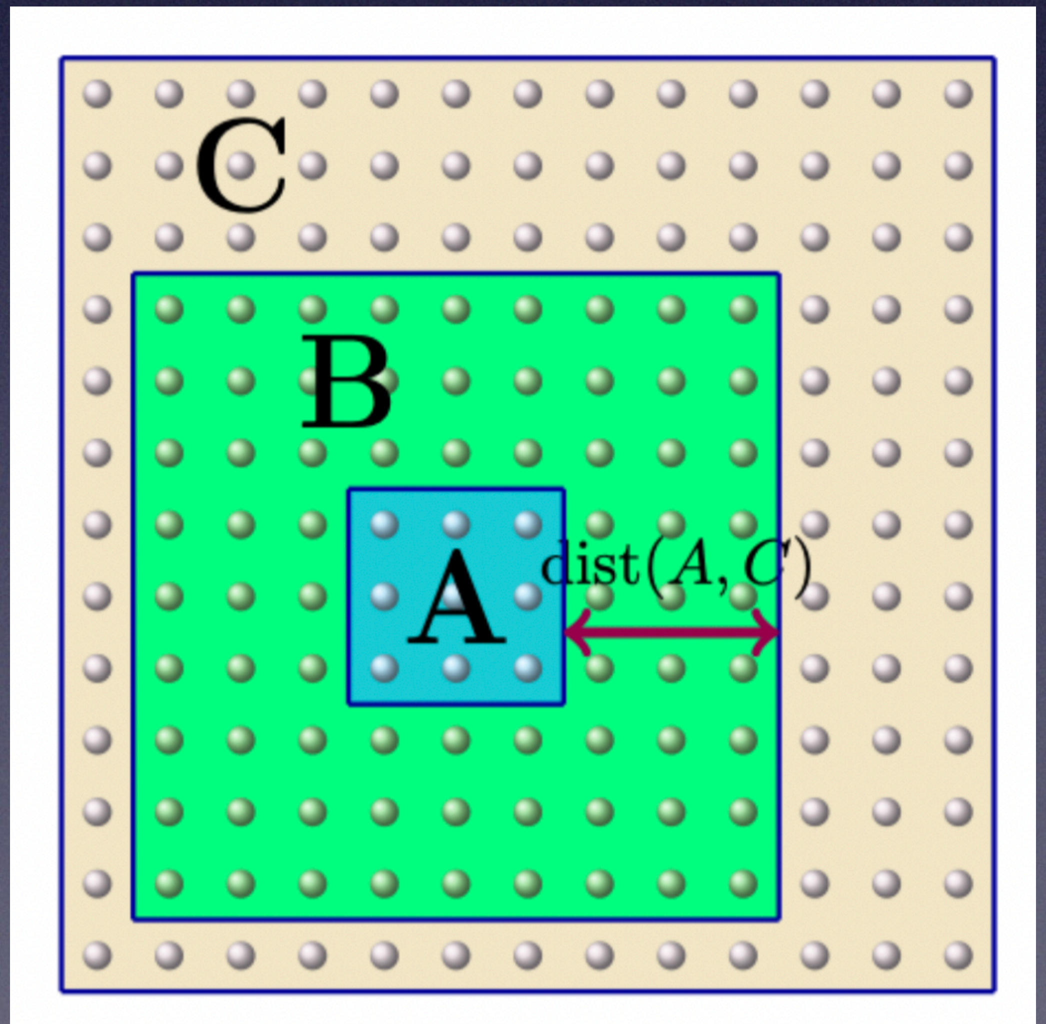
Local indistinguishability

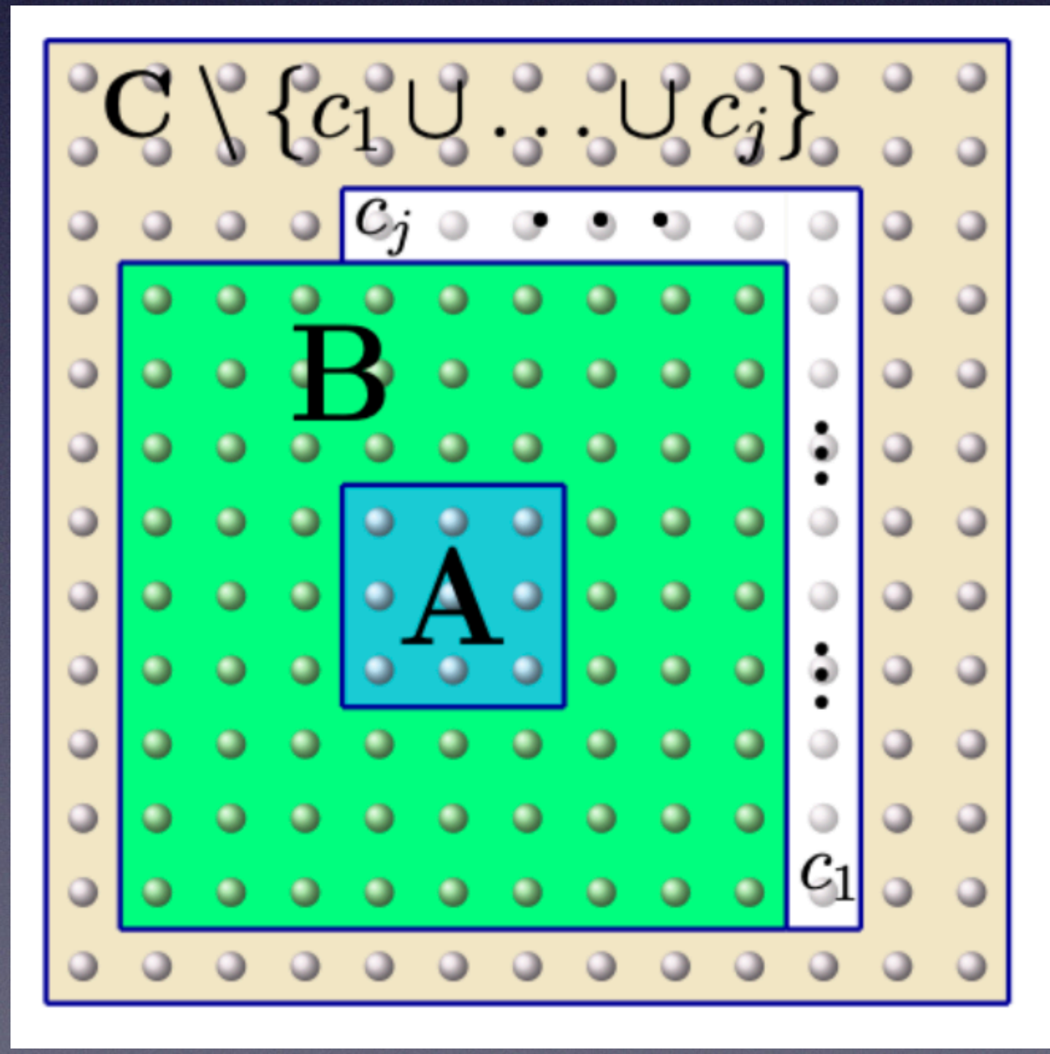
$$\left|\mathrm{Tr}_{ABC}(\rho^{\Lambda}\, O_A) - \mathrm{Tr}_{AB}(\rho^{AB}\, O_A)\right|$$
$$\leq K_{LI} \|O_A\| f_{LI}(A,C)\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$$

[Brandao-Kastoryano, '19]

Local Perturbations Perturb Locally (LPPL)

$$\left|\mathrm{Tr}_{ABC}(\rho^{\Lambda}\, O_A) - \mathrm{Tr}_{ABC}(\widetilde{\rho}^{\Lambda}\, O_A)\right|$$
$$\leq K_{LPPL} \|O_A\| f_{LPPL}(A,C)\, \mathrm{e}^{c\beta \|V_C\|}\, \mathrm{e}^{-\alpha_{LPPL} d(A,C)}$$





- Idea: Remove site by site in the boundary (from the interactions in the Hamiltonian)
- Use QBP and exp. decay of covariance to show that the change at each step is small

This requires uniform exponential decay of covariance!

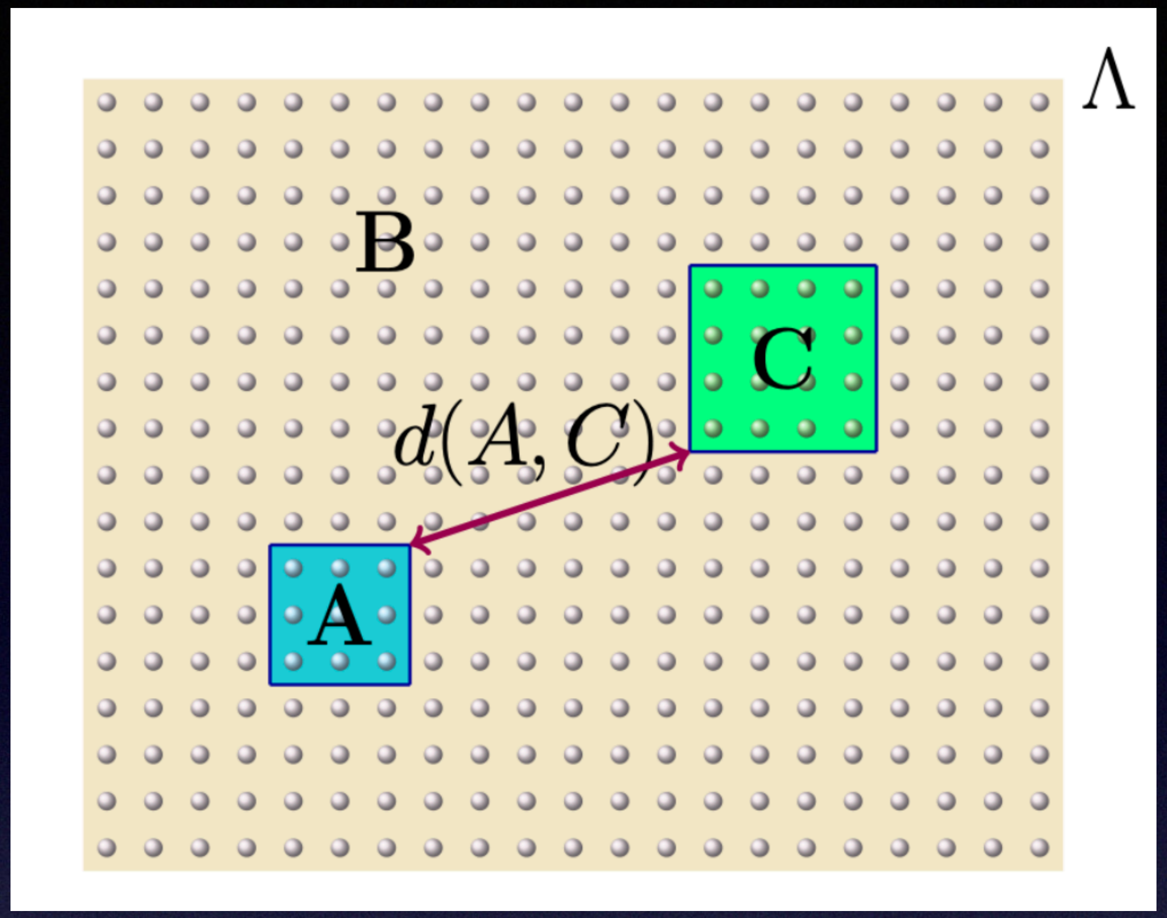
[Kuwahara, '24] Otherwise, the scaling is not good enough

Similar idea in [Onorati et al., '23] for learning of Hamiltonians

Strong decay of correlations for Gibbs states in any dimension (Bluhm, C., Pérez-Hernández, '24)

Effective Hamiltonian → **Equivalence of several measures of correlations (covariance, mutual information, mixing condition)**

Decay of correlations (Covariance)

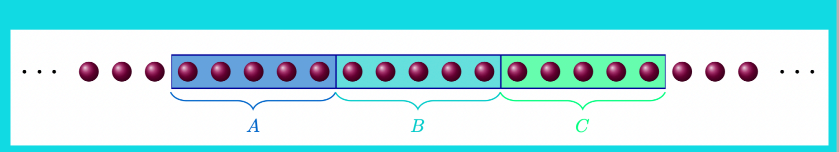
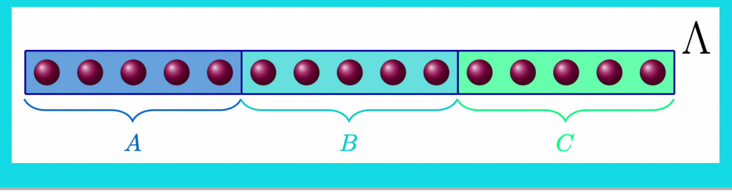


- Lattice $\Lambda \subset\subset \mathbb{Z}^D, \quad \Lambda = ABC$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$, family of Hamiltonians $(H_\Lambda)_{\Lambda \subset\subset \mathbb{Z}^D}$
- Gibbs state $\rho^\Lambda := \frac{e^{-\beta H_\Lambda}}{\mathrm{Tr}[e^{-\beta H_\Lambda}]}$

Exponential decay covariance?

$$\mathrm{Cov}_{\rho^\Lambda}(A,C) = \sup_{\|O_A\|=\|O_C\|=1} |\mathrm{Tr}[\rho^\Lambda O_A O_C] - \mathrm{Tr}[\rho^\Lambda O_A]\mathrm{Tr}[\rho^\Lambda O_C]| \leq K_{Cov} f_{Cov}(A,C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$

1D, translation-invariant Hamiltonian

	Infinite chain 	Finite chain 
Finite range	$\beta > 0$ [Araki, '69]	$\beta > 0$ [Bluhm, C. Pérez-Hernández, '22]
Short range	$\beta < \beta_1$ [Pérez-García, Pérez-Hernández, '23]	$\beta < \beta_1$ [C., Moscolari, Teufel, Wessel, '23]

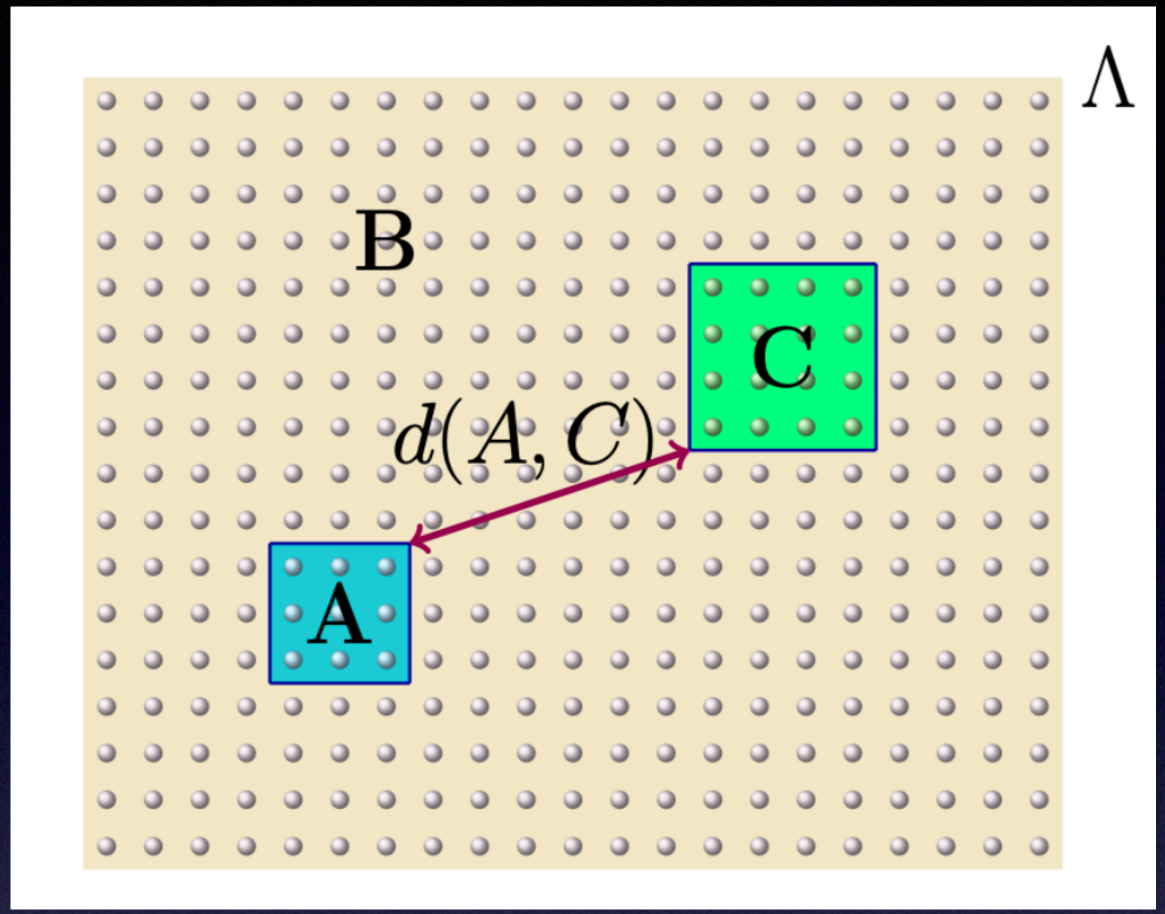
$\beta_1 \to \infty$ for finite range, $f_{Cov}(A,C) = 1$

High D, high-enough temperature

$$\beta < \beta_*$$

Finite range	[Kliesch et al., '14] $f_{Cov}(A,C) = \mathcal{O}(\|\partial A\|, \|\partial C\|)$
Short range	[Fröhlich-Ueltschi, '15] $f_{Cov}(A,C) = \mathcal{O}(\|A\|\|C\|)$

Decay of Mutual Information



- Lattice $\Lambda \subset\subset \mathbb{Z}^D$, $\Lambda = ABC$
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- Family of Hamiltonians $(H_\Lambda)_{\Lambda \subset\subset \mathbb{Z}^D}$

Mutual information $\rho^\Lambda \equiv \rho$

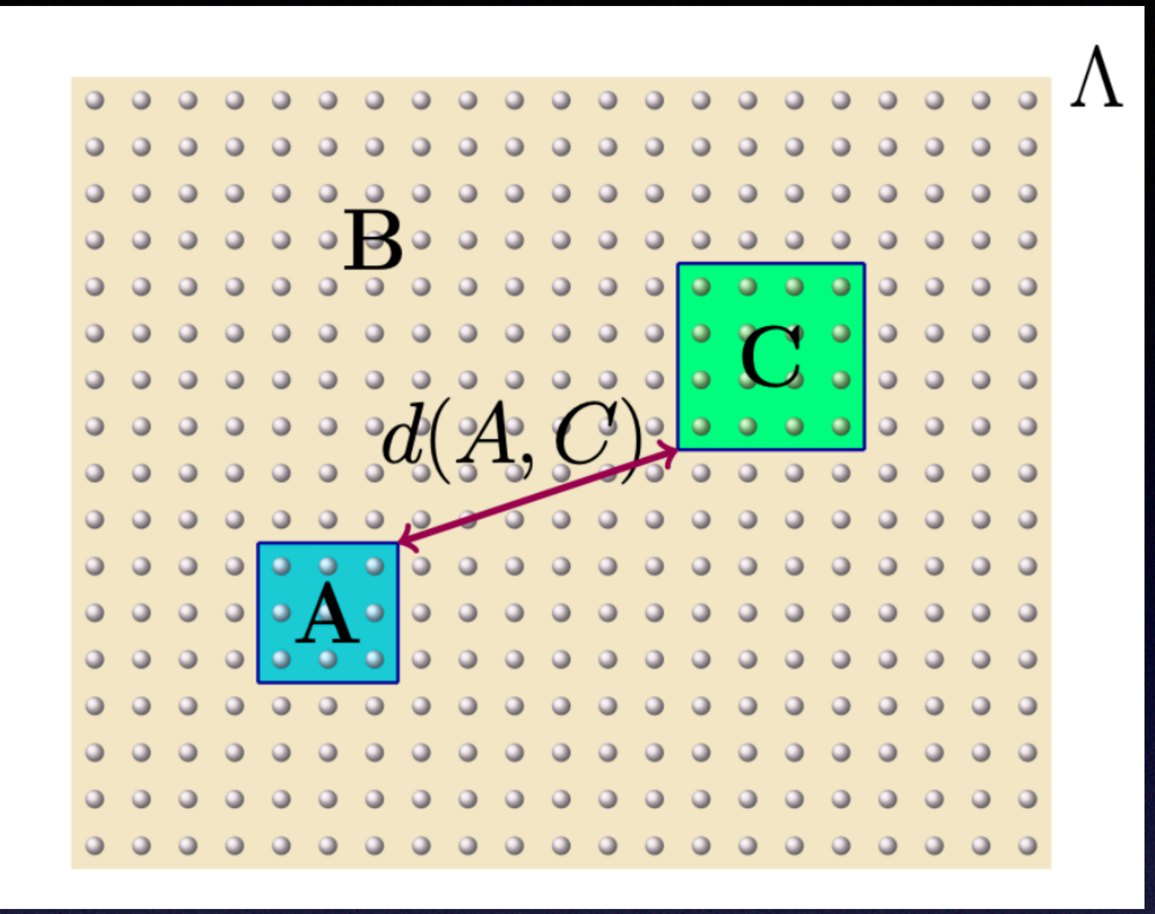
$$I_\rho(A:C) = D(\rho_{AC} \| \rho_A \otimes \rho_C) = \mathrm{Tr}[\rho_{AC}(\log \rho_{AC} - \log \rho_A \otimes \rho_C)]$$

Exponential decay with $d(A,C)$?

$$I_{\rho^\Lambda}(A:C) \leq K_{MI} f_{MI}(A,C)\, \mathrm{e}^{-\alpha_{MI} d(A,C)}$$

(for $K_{MI} > 0$, $\alpha_{MI} > 0$ and $f_{MI}(A,C)$ depending on the size of A and C, but uniform in Λ)

Decay of Mutual Information



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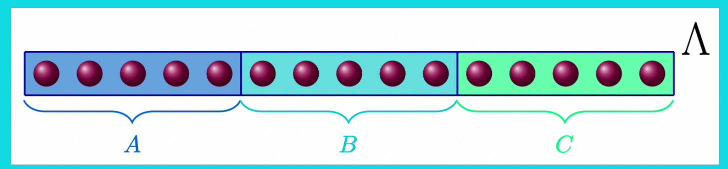
$$I_\rho(A:C) = \mathrm{Tr}[\rho_{AC}(\log\rho_{AC} - \log\rho_A \otimes \rho_C)] \leq K_{MI} f_{MI}(A,C)\, \mathrm{e}^{-\alpha_{MI} d(A,C)}$$

1D, translation-invariant Hamiltonian

	Finite chain
Finite range	$\beta > 0$ [Bluhm, C. Pérez-Hernández, '22] (QIP2022)
Short range	?

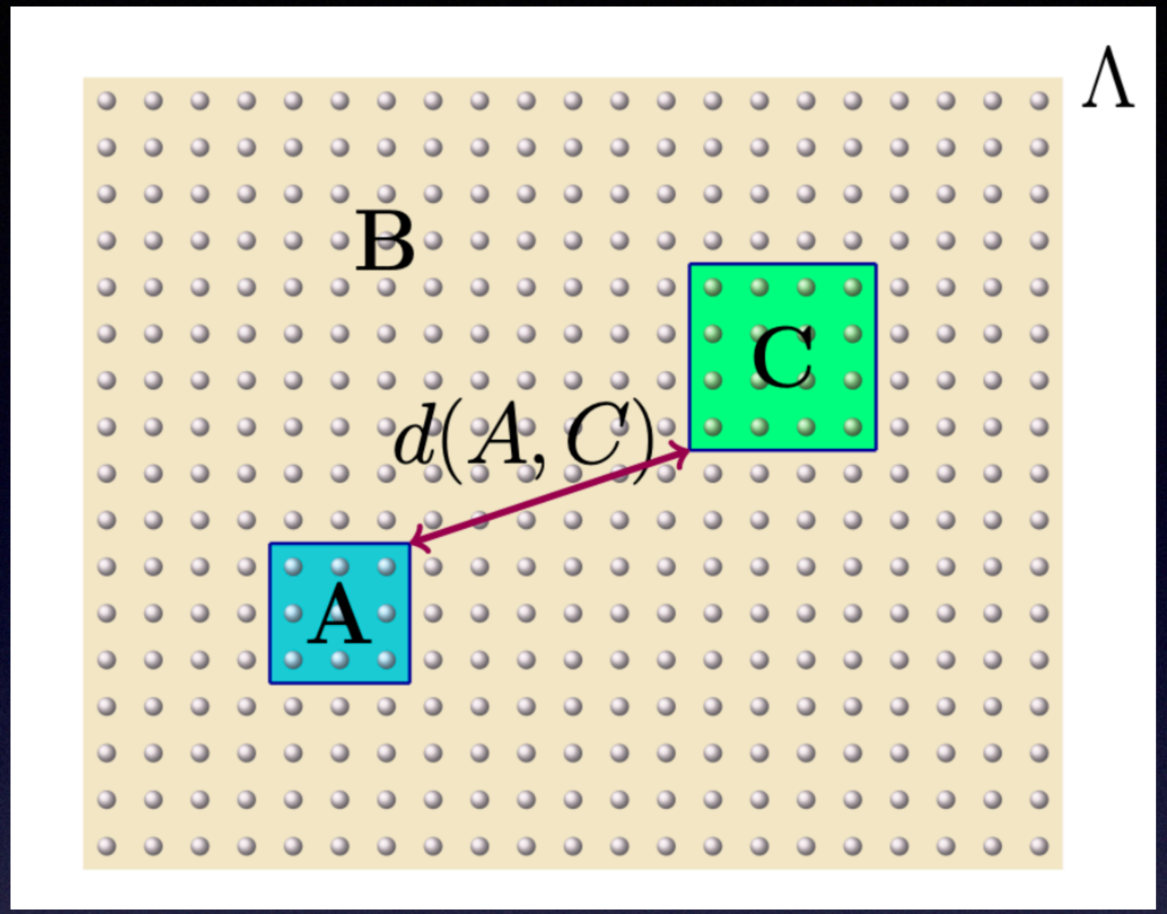


High D, high-enough temperature

$\beta < \beta_*$

Finite range	[Kuwahara et al., '20] ?
Short range	?

Mixing Condition



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  - Family of Hamiltonians $(H_\Lambda)_{\Lambda \subset\subset \mathbb{Z}^D}$

Mixing condition $\rho^\Lambda \equiv \rho$

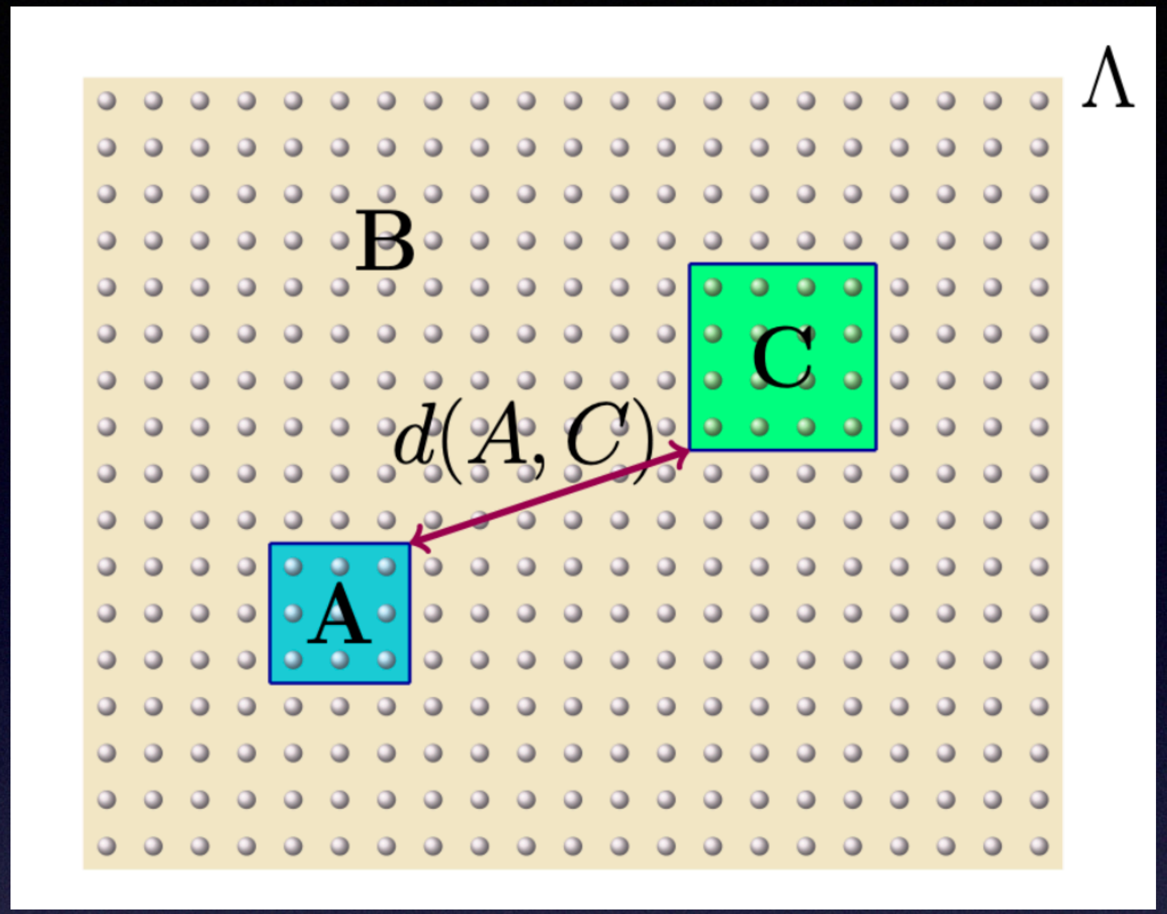
$$\|\rho_{AC}\rho_A^{-1} \otimes \rho_C^{-1} - \mathbf{1}\| \leq K_{MC} f_{MC}(A,C)\, \mathrm{e}^{-\alpha_{MC} d(A,C)}$$

(for $K_{MC} > 0$, $\alpha_{MC} > 0$ and $f_{MC}(A,C)$ depending on the size of A and C, but uniform in Λ)

Instrumental in the proof of Modified Logarithmic Sobolev Inequalities
(and thus rapid mixing of Lindbladians)

[Kochanowski, Alhambra C., Rouzé, '24]

Mixing Condition



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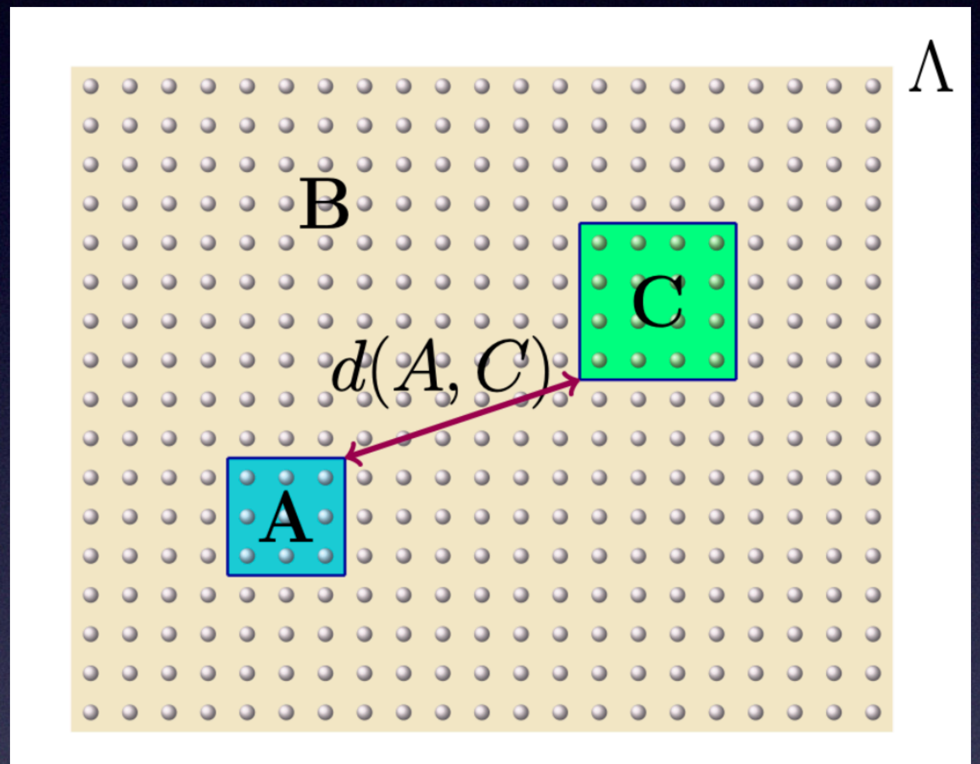
(for $K_{MC} > 0$, $\alpha_{MC} > 0$ and $f_{MC}(A,C)$ depending on the size of A and C, but uniform in Λ)

→ Holds for 1D translation-invariant Hamiltonians, at any inverse temperature [Bluhm, C. Pérez-Hernández, '22]

↓ Instrumental in the proof of Modified Logarithmic Sobolev Inequalities (and thus rapid mixing of Lindbladians) [Bardet et al, '24] [Kochanowski, Alhambra C., Rouzé, '24]

Previous results: relation



- Covariance $\mathrm{Cov}_\rho(A,C) = \sup_{\|O_A\|=\|O_C\|=1} |\mathrm{Tr}[\rho\, O_A O_C] - \mathrm{Tr}[\rho\, O_A]\mathrm{Tr}[\rho\, O_C]|$
- Mutual information $I_\rho(A:C) = \mathrm{Tr}[\rho_{AC}(\log\rho_{AC} - \log\rho_A \otimes \rho_C)]$
- Mixing condition $\|\rho_{AC}\rho_A^{-1} \otimes \rho_C^{-1} - \mathbf{1}\|$

Relation

$$\frac{1}{2}\,\mathrm{Cov}_\rho^2(A,C) \leq I_\rho(A:C) \leq \|\rho_{AC}\rho_A^{-1} \otimes \rho_C^{-1} - \mathbf{1}\|$$

Previous results: 1D translation-invariant, finite-range

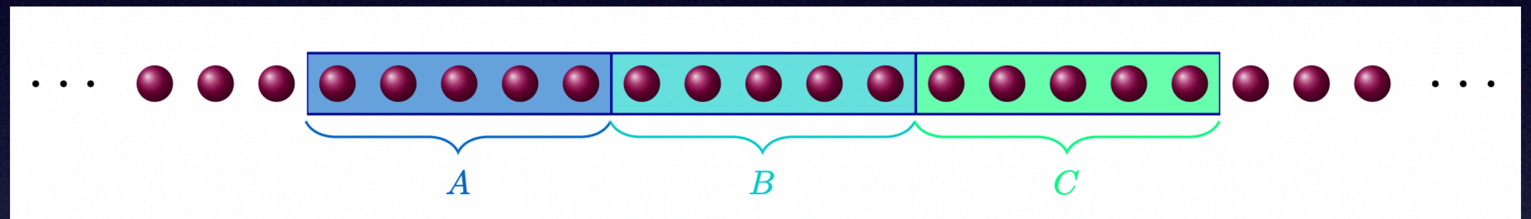
1D, translation-invariant

$\beta < \infty$

[Araki, '69] finite-range

Exponential decay covariance (infinite chain)

$$\mathrm{Cov}_{\rho^\Lambda}(A,C) \leq K_{Cov}\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$



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[Bluhm, C. Pérez-Hernández, '22]

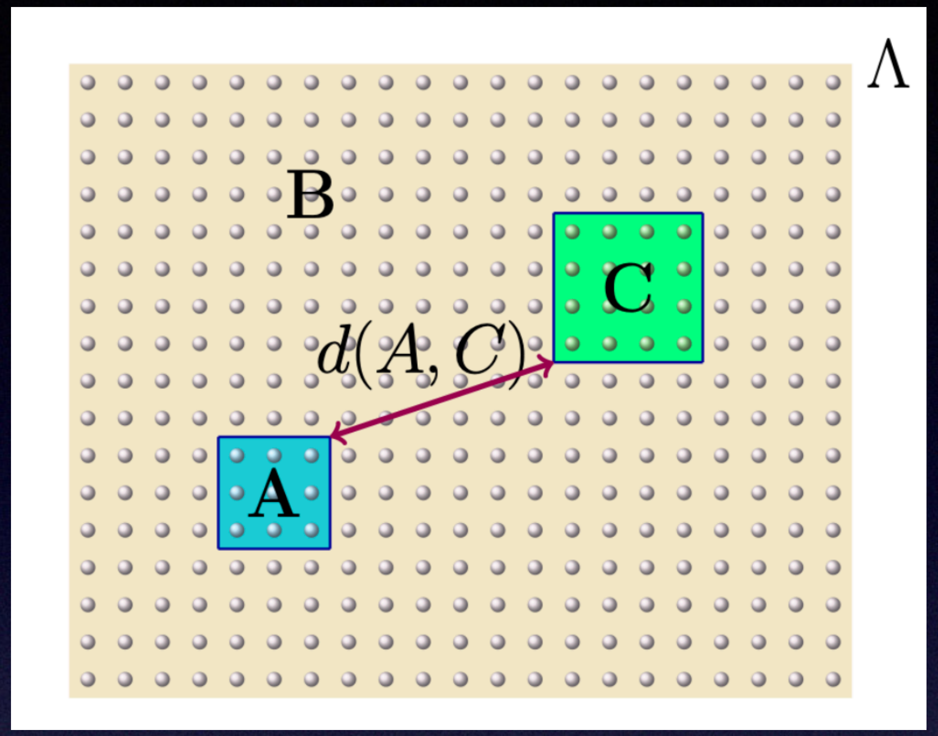
Exp. decay mutual information

$$I_\rho(A:C) \leq K_{MI}\, \mathrm{e}^{-\alpha_{MI} d(A,C)}$$

Mixing condition

$$\|\rho_{AC}\rho_A^{-1} \otimes \rho_C^{-1} - \mathbf{1}\| \leq K_{MC}\, \mathrm{e}^{-\alpha_{MC} d(A,C)}$$

Previous results: High dimension, finite-range, commuting



High dimension, commuting

$\beta < \beta_*$

[Kliesch et al., '14] finite-range

Exponential decay covariance

$$\mathrm{Cov}_{\rho^\Lambda}(A,C) \le K_{Cov}\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$

Exp. decay mutual information

$$I_\rho(A:C) \le K_{MI}\, \mathrm{e}^{-\alpha_{MI} d(A,C)}$$

Mixing condition

$$\|\rho_{AC}\rho_A^{-1}\otimes\rho_C^{-1} - \mathbf{1}\| \le K_{MC}\, \mathrm{e}^{-\alpha_{MC} d(A,C)}$$

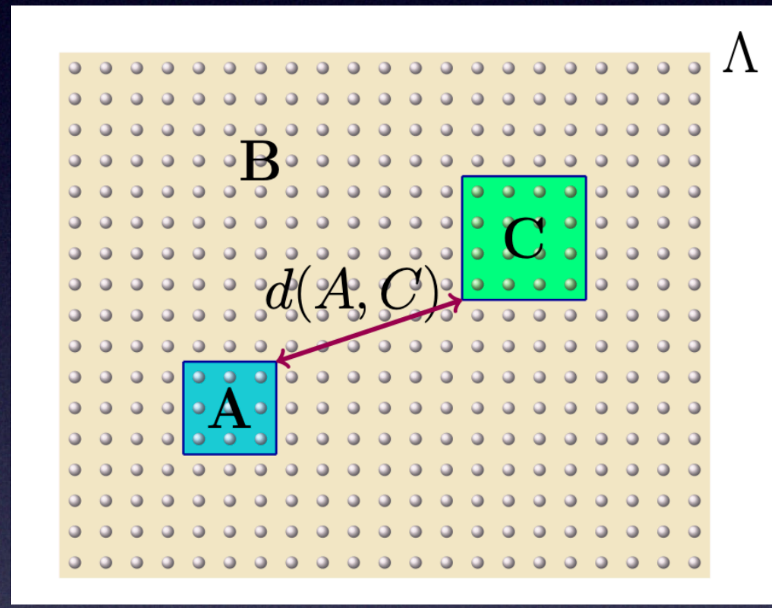
[Kochanowski, Alhambra C., Rouzé, '24]

HERE: High dimension, short-range

High dimension, short-range

$\beta < \beta_*$

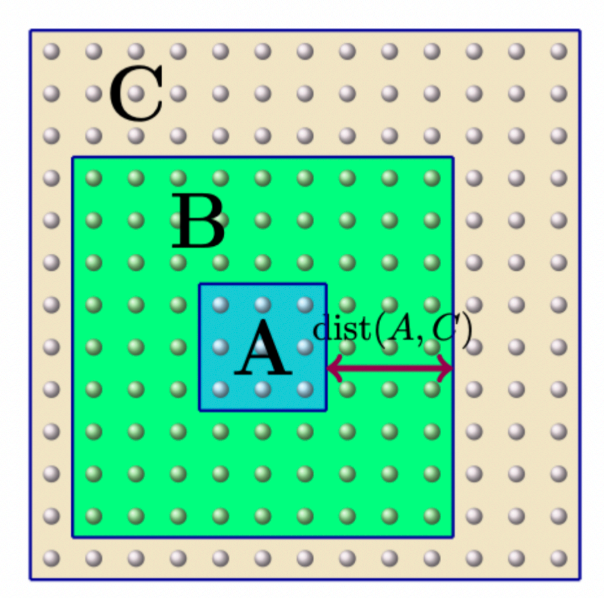
[Fröhlich-Ueltschi, '15] short-range



Exponential decay covariance

$\mathrm{Cov}_{\rho^\Lambda}(A,C) \leq K_{Cov} f_{Cov}(A,C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$

QBP + LR bounds



Local indistinguishability

$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{AB}(\rho^{AB} O_A)\right|$
$\leq K_{LI} \|O_A\| f_{LI}(A,C)\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$

Assuming Effective Hamiltonian

Mixing condition

$\|\rho_{AC}\rho_A^{-1} \otimes \rho_C^{-1} - \mathbf{1}\| \leq K_{MC} f_{MC}(A,C)\, \mathrm{e}^{-\alpha_{MC} d(A,C)}$

Exp. decay mutual information

$I_\rho(A:C) \leq K_{MI} f_{MI}(A,C)\, \mathrm{e}^{-\alpha_{MI} d(A,C)}$

[Bluhm, C. Pérez-Hernández, '24]

Requirement proof: Effective Hamiltonian

- Lattice $\Lambda \subset\subset \mathbb{Z}^D$
- Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} H_X$

Effective Hamiltonian (strong)

$$\widetilde{H}_\Lambda^{L,\beta} := -\frac{1}{\beta} \log\left(\mathbb{E}_L[e^{-\beta H_\Lambda}]\right) = \sum_{X \subset \Lambda} \widetilde{\Phi}_X^{L,\beta}$$

Effective Hamiltonian [Kuwahara et al. '20] [C., Bluhm, Pérez-Hernández '24] (short-range, commuting)

$$\widetilde{H}_\Lambda^{L,\beta} := -\frac{1}{\beta} \log\left(\mathrm{tr}_{L^c}[e^{-\beta H_\Lambda}] \otimes \mathbf{1}_{L^c}\right) + \frac{1}{\beta}\log[Z_{L^c}]\mathbf{1} = \sum_{X \subset \Lambda} \widetilde{\Phi}_X^{L,\beta}$$

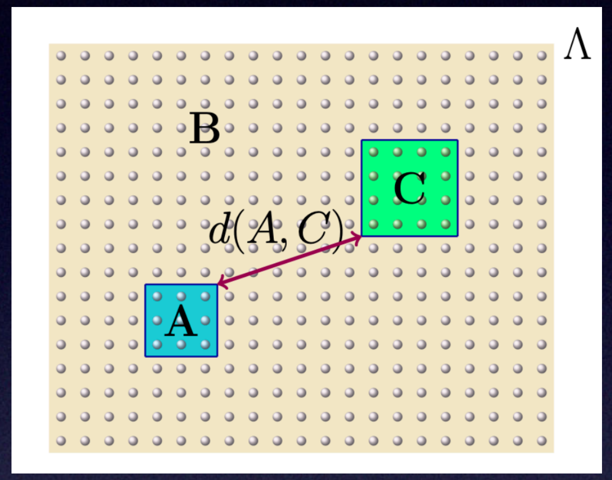
- $\widetilde{\Phi}_X^{L,\beta}$ is supported in $X \cap L$,
- if $X \subset L^c$ then $\widetilde{\Phi}_X^{L,\beta} = 0$,
- if $X \subset L$ then $\widetilde{\Phi}_X^{L,\beta} = H_X$,
- if $L \subset L'$ and $X \cap (L' \setminus L) = \emptyset$ then $\widetilde{\Phi}_X^{L,\beta} = \widetilde{\Phi}_X^{L',\beta}$.

Summary results

We have shown: Several properties of locality and decay of correlations on Gibbs states are equivalent!

Exponential decay covariance

$$\mathrm{Cov}_{\rho^\Lambda}(A,C) \le K_{Cov} f_{Cov}(A,C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$



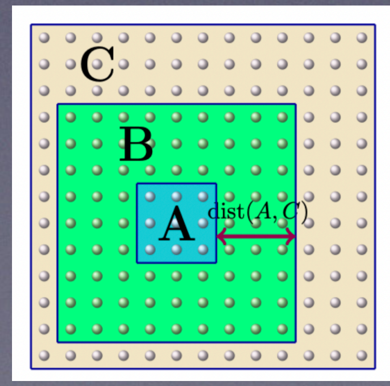
[C., Moscolari, Teufel, Wessel, '23]

Local Perturbations Perturb Locally (LPPL)

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{ABC}(\widetilde{\rho}^\Lambda O_A)\right| \le K_{LPPL}\|O_A\| f_{LPPL}(A,C)\, \mathrm{e}^{c\beta\|V_C\|}\, \mathrm{e}^{-\alpha_{LPPL} d(A,C)}$$

Local indistinguishability

$$\left|\mathrm{Tr}_{ABC}(\rho^\Lambda O_A) - \mathrm{Tr}_{AB}(\rho^{AB} O_A)\right| \le K_{LI}\|O_A\| f_{LI}(A,C)\, \mathrm{e}^{-\alpha_{LI} d(A,C)}$$



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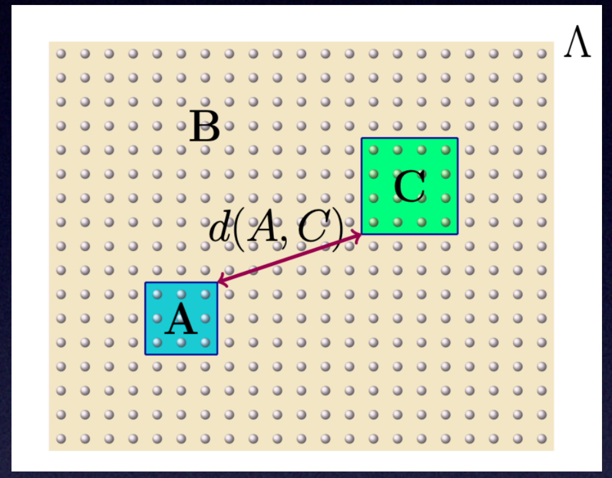

Exp. decay mutual information

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Mixing condition

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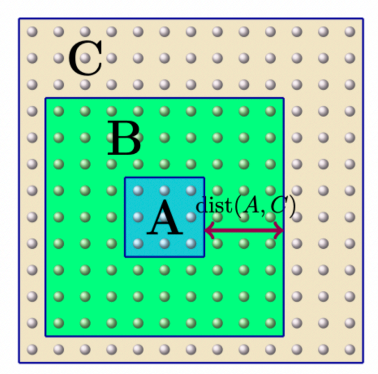

Summary results

1D, translation-invariant

$\beta < \beta_1$

[Araki, '69] finite-range

[Pérez-García, Pérez-Hernández '23] short-range

High dimension

$\beta < \beta_*$

[Kliesch et al., '14] finite-range

[Fröhlich-Ueltschi, '15] short-range

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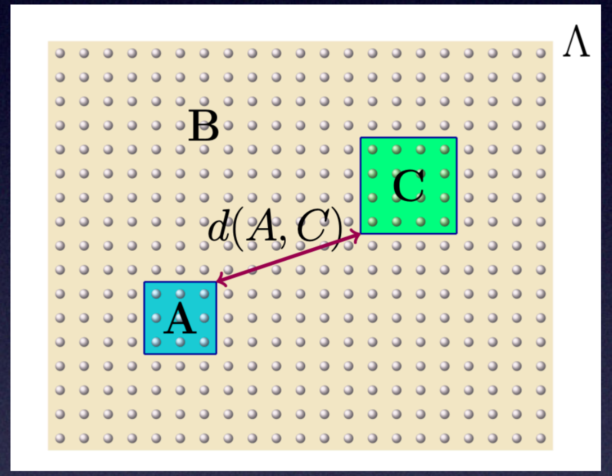

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[Bluhm, C. Pérez-Hernández, '24]

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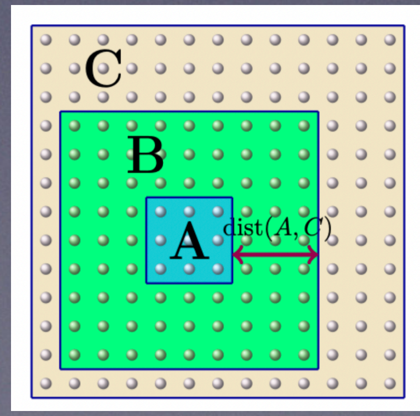


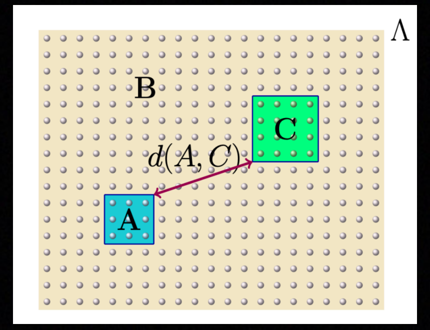
Open Problems

- Existence of the effective Hamiltonian?
- Long-range interactions?
- More applications?
- Infinite dimensions? In particular, for the Quantum Belief Propagation
- Extend results to 0 temperature (ground states)?
- Open systems?

And many more!

Exponential decay covariance

$$\mathrm{Cov}_{\rho^\Lambda}(A,C) \le K_{Cov} f_{Cov}(A,C)\, \mathrm{e}^{-\alpha_{Cov} d(A,C)}$$



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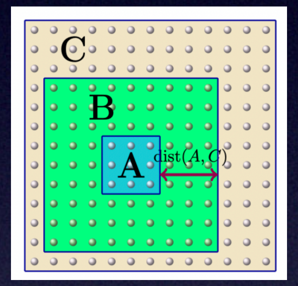

Thanks for your attention!

(And also to my coauthors!)